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PROMOTING MATHEMATICAL DISCUSSION: UNPACKING THE PEDAGOGY OF AN EARLY CHILDHOOD EDUCATOR

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PROMOTING MATHEMATICAL DISCUSSION: UNPACKING THE PEDAGOGY OF AN EARLY CHILDHOOD EDUCATOR

by

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Promoting Mathematical Discussion:
Unpacking the Pedagogy of an Early Childhood Educator

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This four-month-long qualitative case study looks closely at how one kindergarten teacher tried to help young children have more mathematical discussions. To discover and more deeply understand a kindergarten teacher's ways of thinking about and facilitating mathematical discussion as part of everyday mathematical instruction, data was collected through classroom observations, semi-structured interviews, and various forms of documentation. Through data analysis in the constant-comparative method, this study found that intensive-discussion mathematics lessons could be accomplished through two pedagogical roles of the teacher. The first was creating a respectful learning environment to motivate children's participation in mathematical discussion. The second was scaffolding student discussions to construct their own knowledge in the path of their

mathematics learning. The analysis detailed here also revealed that successes and failures of discussion-intensive mathematics lessons depend on the teacher's ability to overcome challenges she encounters while integrating mathematical discussion into her everyday lessons.

The presented examples and descriptions in this study offer significant implications for early childhood teachers. This is particularly true for those who care about their young students' mathematical development, yet either struggle to develop trusting classroom communities or do not know how to facilitate mathematical discussion. This study also provides insights into how teacher educators can help preservice teachers develop a profound understanding of mathematics teaching and learning. This highly influences their moment-by-moment decision-making to appropriately scaffold young children's talk and learning. It offers implications for administrators about how to support early childhood teachers' growth, learning, and their practices in teaching mathematics.

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Chapter I. Introduction

> The purpose of whole-class discussion is to provide students with practice in mathematical reasoning that will further their mathematical learning. To accomplish this, the focus is on the students' ideas, not on the correctness of their answers. This does not mean that we are advising teachers to deemphasize correct answers and mathematical truth. In our view, the ultimate goal is for students to achieve mathematical power through precision, accuracy, insight, and reliable reasoning. However, we have found that it's important for students to have opportunities to practice their reasoning in discussions without an immediate focus on correct answers. (Chapin, O'Connor, & Anderson, 2003, p. 18)

PROBLEM STATEMENT

For more than three decades, many educational researchers, drawing from the socio-constructivists' psychological viewpoints of learning (Erickson, 1996; Vygotsky, 1978; Wertsch, 1991), have been greatly concerned with language and social interaction as core components of student mathematical thinking and learning (e.g., Cobb, Wood, & Yackel, 1993; O'Connor & Michaels, 1996; O'Connor, 1998; Lampert & Blunk, 1998). Their groundwork underlies much of the last wave of reform in mathematics education. The National Council of Teachers of Mathematics (NCTM, 2000, 2006), bringing definition to the reform movement in North America, has consistently recommended that teachers foster productive conversations that enable students to do certain things—to coherently organize their mathematical thinking, to communicate their mathematical ideas with one another other, to analyze their mathematical strategies, and to develop their conceptual understanding of mathematics. This recommendation has stimulated a new model of social interaction in the mathematics classroom in which "talking about math becomes acceptable, indeed essential in the classroom, and mathematical discussion, explanation, and defense of ideas become defining features of a quality mathematical experience" (Walshaw & Anthony, 2008, p. 516).

The intersection of researchers' continued attention to classroom discussion, recent reform efforts in mathematics education, and a growing concern about the mathematical growth of students have inspired many mathematics educators to pursue a number of research lines, including studies of the value of discussion in the classroom (e.g., McClain & Cobb, 2001; O'Connor, 2001), participation frameworks in mathematical discourse (e.g., Cobb, Wood, & Yackel, 1993; O'Connor & Michaels, 1996), beneficial effects of repeated participation in discussing mathematics (e.g., Baxter, Woodward, & Olson, 2001; Lubienski, 2002), and instructional strategies for students' engagement in mathematical discussion (e.g., Chapin, O'Connor, & Anderson, 2003; Lampert & Blunk, 1998). The assumption of these studies is that "just getting students to talk [is] not enough; what they [need to talk] about mattered" (Franke, Kazemi, & Battey, 2007, p. 232). Through listening respectfully to and critiquing others' ideas, and through arguing and defending their own positions (Walshaw & Anthony, 2008), students need to "purposeful[ly] talk on a mathematical subject in which there are genuine pupil contributions and interaction[s]" (Pirie & Schwarzenberger, 1988, p. 461). Accordingly, in order to contribute to the enhancement of students' motivation, attitudes, and achievements in mathematics, the teacher must successfully orchestrate students' purposeful participation in mathematical discussion (Stein et al., 2008).

The research suggests that helping students have more classroom discussions is applicable not only to secondary mathematics education but also to primary mathematics education—even lower elementary grade levels. For instance, O'Connell and O'Connor (2007) argue that teacher-facilitated classroom discussions are vital ways to motivate and engage pre-k to second grade students in learning mathematics. Empson (2003, p. 305) emphasizes a teacher's role in using "tasks that elicited the students' prior understanding," creating "a variety of participant framework," and providing multiple "opportunities for identify-enhancing interactions." These contribute to the first-grade low-performing students' participation in classroom discourse. Moreover, Pierson et al. (2007) explore how one kindergarten teacher modifies her teaching practices, as she attempts to deliberately engage research-based discourse practices to support student mathematical understanding. They reveal that the core factors of the teacher's success at facilitating mathematical discourse are "deliberate reflection and planning, flexibility so student ideas can contribute to the flow of the lesson, and appropriate support from collaborating researchers" (p. 3). These studies propose that the teachers should have a major role in promoting productive discussions in ways that enable young children to construct mathematical understanding.

However, despite these educational studies, the teacher-facilitated mathematical discussions are not widely welcomed in kindergarten. Many kindergarten mathematics classrooms still use traditional, transmission-oriented instruction (e.g., a typical sequence of teacher initiation, student response, and teacher evaluation (Whitin & Whitin, 2003). The substantial gaps between what research suggests about mathematical discussion and what actually happens in a kindergarten classroom are influenced by various problems. For example, Skipper and Collins (2003) argue that early childhood teachers have a misconception that “play is the only important and developmentally appropriate approach for young children” (p. 422). Even when teachers understand the importance of discussion in mathematics, it is often difficult to put into practice. Despite teachers’ efforts to learn the importance of verbal and social interaction and to recognize the teacher’s role in fostering discussions, many early childhood teachers may still be confused and anxious about how to initiate mathematical talks and facilitate classroom discussions (Lee & Ginsburg, 2009). They may have a limited focus on informal strategies to enhance kindergarteners’ use of mathematical communication during the whole day, such as serving as facilitators during center time and connecting classroom routines to mathematics (Cooke & Buchholz, 2005). Furthermore, increasing the emphasis on academic learning and accountability appears to make teachers favor a highly scripted approach or teacher-directed instruction focused on rote learning and memorization (Goldstein, 2007; Skipper & Collins, 2003). It also causes many early childhood teachers to feel that classroom discussions may be ineffective for preparing young children for standardized achievement mathematics tests (Jung & Reifel, 2011).

These findings show the importance of exploring what it is that kindergarten teachers actually do to deal with mathematical discussions, “through a broadened view of the individual [teachers] in [their own] context[s]” (Schallert & Martin, 2003, p. 42). What has emerged is a growing though still small body of empirical studies exploring this issue at the kindergarten level (e.g., Cooke & Buchholz, 2005; O’Connell & O’Connor, 2007; Pierson et al., 2007; Skipper & Collins, 2003). The previous studies have not yet studied how teachers in their classroom spaces create mathematical discussion nor have they documented how teachers explain the pedagogical importance of mathematical discussion. The collective limitations of these studies emphasize the crucial need for qualitative research on more deeply understanding of a kindergarten teacher's ways of thinking about and facilitating mathematical discussion within his/her own classroom. This dissertation study builds on this prior knowledge but adds detailed data about how a teacher specifically explains her own thoughts, ideas, assumptions and inquiries about classroom discussions as part of everyday mathematical instruction. Also, this study shows how a teacher makes decisions to facilitate classroom discussions moment-by-moment within the constraint of the current school systems.

RESEARCH QUESTIONS OF THE STUDY

For the purpose of investigating a kindergarten teacher's role in mathematical discussion, the research was guided by the following two questions:

1. How does a kindergarten teacher conceptualize his/her role in mathematical discussion for young children?
2. How does a kindergarten teacher orchestrate young children's participation in mathematical discussion during mathematics lessons?

To this end, I specifically used the term *mathematical discussion*, defined as "purposeful talk on a mathematical subject in which there are genuine pupil contributions and interaction[s]" (Pirie & Schwarzenberger, 1988, p. 461).

In order to achieve an intensive and holistic understanding of a particular kindergarten teacher's ways of thinking about and facilitating mathematical discussion within his/her own classroom, I engaged in a qualitative case study by investigating one participant as a single unit (Stake, 1995). Because a single-case study was used, I was able to investigate the teacher's ideas and practices in detail and could see, for example, the words and phrases she used to begin class discussions over a number of days and weeks. For this, I carefully selected one experienced kindergarten teacher, using purposeful sampling (Patton, 1990) as the process to determine the criteria that would guide the selection as a good case "from which the most can be learned" (Merriam, 1998, p. 61). To explore a single case in depth, I collected multiple sources of data: observing and audio-recording mathematics lessons, taking fieldnotes and reflexive journals, semi-structured interviewing and informal conversations with the teacher, and documenting in terms of a kindergarten teacher's role in mathematical discussion.

From an analysis of classroom observations and interviews with one kindergarten teacher, I found that she conceptualized and tried to fulfill three main roles in order to facilitate mathematical discussion. The three roles are (1) *creating a respectful learning environment*, (2) *scaffolding student discussions*, and (3) *overcoming challenges to mathematical discussion*. In chapters Five, Six and Seven, I spend time on each role respectively, addressing each of these roles both in terms of how the teacher thought about each role (research question 1) and how she fulfilled each role in the classroom (research question 2).

SIGNIFICANCE OF THE STUDY

Through a qualitative case study of an experienced kindergarten teacher's role in mathematical discussion, this research contributes to a better understanding of how to promote young children's engagement in mathematical discussion that can assist with the development of their confidence and ability in mathematics. This exploration also provides early childhood teachers with practical insights into how to integrate pedagogical strategies to facilitate mathematical discussion into their own math lessons within today's public school systems.

DISSERTATION OVERVIEW

In this Chapter One, I have provided a rationale for the use of mathematical discussion in teaching and learning mathematics for young children, established the accompanying research questions on the kindergarten teacher's role in orchestrating young children's participation in mathematical discussion, and described the potential significance of the study.

Chapter Two describes in detail the literature that informed my theoretical framework to guide this study, and Chapter Three reviews empirical studies in terms of the teacher's role in mathematical discussion. Chapter Four outlines the research methodology I use, by illustrating research design, by discussing the participant, data collection, data analysis, procedure for establishing trustworthiness, ethical considerations, and researcher positionality, and by concluding with the study's timeline.

Chapters Five, Six, and Seven respectively outline the findings drawn, from an analysis of the classroom observations and interviews with one kindergarten teacher, concerning her three core roles in (1) creating the respectful learning environment, (2) scaffolding student discussions, and (3) overcoming challenges to mathematical discussion. Finally, Chapter Eight contains major discussion points derived from the findings by responding to the research questions. It also illustrates implications for early childhood teachers and educators, the limitations of this study, and suggestions for further study.

Chapter II. Theoretical Framework

Chapter Two outlines the literature that informed my theoretical framework to guide this study in the following five sections. I first illustrate the language viewpoint of socio-constructivism as a theoretical lens to understand the notion of social interaction and dialogue as key mediated tools in the dynamic process of student learning. Second, I briefly draw on Goffman's (1981) conception of the participation framework as a point of departure to understand how participant roles in discussions contribute to children's learning. Third, I describe my conceptual understanding of young children's mathematical learning. Fourth, I explain how to look at a teacher's constructions of knowledge and practices within the context. Last, I point out the importance of understanding a teacher and students' cultural, ethnic, and linguistic backgrounds that are essential to explore how they interact and communicate in mathematical discussions. Those five components of my theoretical framework help me explore the two research questions, in terms of how the kindergarten teacher thinks about facilitating mathematical discussion (research question 1, p. 5) and how she actually tries to have mathematical discussions in her classroom (research question 2, p. 5). This framework guides how I analyze and interpret my participant teacher's conceptions, experiences, and decisions in relation to her creation of mathematical discussion for young children. It also allows me to look at the meanings and motives behind the teacher's thoughts and actions in initiating and facilitating mathematical discussion in the kindergarten context.

SOCIO-CONSTRUCTIVIST VIEWS OF THE CLASSROOM DISCUSSION

The work of the socio-constructivists, who are interested in language and its function in an individual's knowledge acquisition and cognitive development, provides a complementary lens for understanding the role of classroom discussion in the process of student learning. This study's theoretical premise is grounded in two key assumptions of socio-constructivist theory: knowledge acquisition in social context and learning mediated through language.

Knowledge acquisition in social contexts

The first key assumption of socio-constructivist theory is that "learning proceeds most effectively within a social context" (Brenner, 1998, p. 153). Vygotsky (1978), who laid down the most significant bases of the socio-constructivist theory, explains the two stages of learners' knowledge construction: "first in the social, later in the psychological, first in relations between people as an interpsychological category, afterwards within the child as an intrapsychological category" (Valsiner, 1987, p. 67). This explication describes how learning is "not tied solely to individual processes" (Wertsch, 1998, p. 23) but always first situated in social interaction with others and then internalized (Schallert & Martin, 2003), so that "the specific structures and processes of intrapsychological functioning can be traced to their genetic precursors on the interpsychological plane" (Wertsch, 1991, p. 89). And yet, this statement of the social nature of learning does not assume that every social activity directly and simply leads to individuals' higher mental functions, such as thinking, voluntary attention, and logical memory (Wertsch, 1991).

Higher mental processes in an individual child's learning occur when socially organized processes fall within his/her zone of proximal development (ZPD) (Vygotsky, 1978).

This zone is defined as "the distance between the actual developmental level, as determined by independent problem solving, and the level of potential development, as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). The ZPD is not static but shifts as the child gains a higher level of thinking and knowledge (Bodrova & Leong, 2007). This sequence of constantly changing zones in social interaction with others can lead to a child's cognitive structure development (Moll, 1990). For instance, what the child did only with assistance yesterday becomes a level of independent performance today; then, as he/she handles more difficult tasks, a new level of assisted performance emerges; this cycle between actual and potential level of development is repeated over and over again until he/she completely constructs a body of knowledge (Bodrova & Leong, 2007). In this regard, Vygotsky (1978) emphasizes the teacher's role in figuring out the potential level of an individual child's development, so that the teacher provides individualized instruction within each child's ZPD (Wertsch, 1991). Further, Lave and Wenger (1991) point out the learner's role in taking on increasing responsibility for participating his/her own learning, so that the learner become an active knowledge constructor in the learning community, beyond the initial stage that the learner participates in simple and low-risk tasks by depending on the assistance of more capable peers or adults.

Learning mediated through language

Another central tenet of a socio-constructivist view is that higher mental functioning in human learning is typically mediated by sign systems (Vygotksy, 1978; Wertsch, 1991). This sign-based mediation includes many forms of semiotic means, such as "language; various systems of counting; mnemonic techniques; algebraic symbol systems; works of art; writing; schemes, diagrams, maps and mechanical drawings; all sorts of conventional signs and so on" (Vygotsky, 1981, p. 137, cited in John-Steiner & Mahn, 1996, p. 193). Among these examples, language is a key-mediating tool in the process of internalization of social interaction in the construction of knowledge (Wertsch, 1991). Vygotsky (1978) argues that a child's learning always springs from social interactions with others, and further his/her understanding is highly mediated through verbal exchanges "when there is a speaker and a listener sharing the same language and rules of communication in a particular context under particular conditions" (Wertsch, 1998, p. 19). That is, social interactions enable children to construct the meaning embedded in speech patterns, written language, and other symbolic knowledge, as well as socially coordinating their actions with others through shared meaning (Moll, 2001). Moreover, verbal exchanges with teachers and more competent peers allow children to develop their own "back and forth processes from thought to word and from word to thought that allow learners to move beyond what would be easy for them to grasp on their own" (Truxaw, Gorgievski, & DeFranco, 2008, p. 58). Accordingly, children's learning cannot be understood apart from their social contexts, and language can be the actual tool to promote individual intellectual development (Moll, 1990) "from interpsychological functioning to intrapsychological functioning, sometimes in nonlinear ways" (Mcintyre, Kyle, & Moore, 2006, p. 40).

Furthermore, in order to encourage higher mental functioning through social interactions, the learner must be guided through appropriate scaffolding within his/her ZPD. According to Wood, Bruner, and Ross (1976), scaffolding is defined as the "process that enables a child or novice to solve a problem, carry out a task, or achieve a goal that would be beyond his unassisted efforts" (p. 96). Dialogue in the classroom is the primary opportunity for scaffolded instruction to enable children to perform at a higher level (Mcintyre, Kyle, & Moore, 2006). Teachers' scaffolding within classroom dialogue may involve "giving hints and clues, rephrasing questions, asking the child to restate what has been said, asking the child what he/she understands, and demonstrating the task" (Bodrova & Leong, 2007). Teachers thus must be sensitive to the child's reaction to the support and assistance provided within the ZPD. In the mathematics classroom, for example, if the student accepts the teacher's support in solving a particular math task, the teacher's instruction falls within this student's ZPD. However, if the student consistently ignores the teacher's help, the type of assistance the teacher provided may be outside this student's zone. Thus, if the student still cannot perform at the higher math level of his/her ZPD as expected, the teacher must rethink and modify teaching strategies for scaffolding. With scaffolding, what the learner does alone is made easier with assistance (Bodrova & Leong, 2007). The conversational process for scaffolding helps teachers look, in a more sensitive way, at how to scaffold each child's learning and development within his/her social and individual zone. These views of language as a mediating function in the mathematics learning process come to place such emphasis on the value of real dialogue and social interactions (e.g., O'Connor & Michaels, 1996) and the role of guided participation (e.g., Rittenhouse, 1998) in mathematics classrooms.

PARTICIPATION FRAMEWORK IN A CONVERSATION SITUATION

Goffman (1981) calls into question the commonly held view on talk as a dyadic model between only two individuals, a speaker and a hearer, in a conversation situation. He argues that this traditional model of talk is not sufficient to analyze the complexity of "the forms of talk sustained within structured social encounters" (Goodwin & Goodwin, 2004, p. 223). To understand conversational process and structure in multiparty interactions, Goffman (1981) introduced the term *participation framework* based on an idea that "[w]hen a word is spoken, all those who happen to be in a perceptual range of the event will have some sort of participation status relative to it" (p. 3). For him, the combination of all persons' participation statues in an encompassing social gathering is defined as "participation framework for that moment of speech" (Goffman, 1981, p. 137).

Using Goffman's (1981) notion of the participation framework, Goodwin (1990) identified not only the interactive process to allocate participants' roles by verbal and nonverbal communication, but also pointed out that there are relational rights and responsibilities that go with participants' own roles in a particular moment for social interactions. These rights and responsibilities for each role of student participation are fostered and scaffolded initially by the teacher (O'Connor & Michaels, 1996), and then are created gradually by the learners through positioning themselves and each other in specific roles within particular classroom activities (Bruner, 1983). That is, the participation in such roles, accompanied by the rights and responsibilities, represents an opportunity for "the increasing autonomy of the learner in stable interactional structures" (Brandt, 1999, p. 310). The learner thus becomes an active meaning maker, so that he/she invents her/his individual knowledge construction through a participation framework in which the learning process takes place directly.

UNDERSTANDING OF YOUNG CHILDREN'S MATHEMATICS LEARNING

My conceptual understanding of young children's mathematical learning has been developed based on guiding constructivists' theories. When I became a kindergarten teacher in Korea, Piaget's (1953) cognitive-constructivism and Vygotsky's (1978, 1981) socio-constructivism were considered the best theories for early childhood education. I also had three years teaching experiences not only creating a child-centered and play-based learning environment, but also supporting young children's learning within these psychological paradigms. Both theoretical stances affected my assumption of how to look at, interpret, and analyze a teacher's efforts to support young children's mathematics learning in classroom discussions.

A tenet of Piaget's constructivism is that knowledge is an individual construction created by the learner as he/she interacts with people and things in the environment (Mistertta, 2008). He described the procedures for these interactions as assimilation and accommodation. Assimilation is the child's action on the environment, and accommodation is the action of the world on the child's thought (Frost, Wortham, &

Reifel, 2008). Piaget believes that play occurs when the child assimilates the world into her/his own conceptions rather than accommodating her/his own perceptions to fit the world. He asserts that play is a reflection of the child's development. Piaget also states that abstract thinking of mathematical ideas is possible only after conceptualization and meaningful understandings have been established (Mistertta, 2008). This view indicates that children as individual meaning-makers develop their own cognitive understandings through play before learning mathematics.

Piaget's cognitive stages have been used as the basis for developmentally appropriate mathematics education for young children. During all four stages, the sensorimotor stage, the preoperational stage, the concrete operational stage, and the formal operational stage in adolescence (Woolfolk, 2004), children engage in certain types of play and develop within certain structures and characteristics of cognition. Also, Piaget's theory about physical knowledge, social-conventional knowledge, and logico-mathematical knowledge is useful for defining math objectives for young children's play (Kamii, Miyakawa, & Kato, 2004). Mathematics grows out of the logico-mathematical knowledge that each child constructs from within, through his/her own ability to think (Kamii & Kato, 2006). While physical and social-conventional knowledge have sources outside the individual, logico-mathematical knowledge, such as classification, seriation, and numbers, consists of mental relationships that originate in each child's head (Kamii & Kato, 2006). This Piagetian point of view about knowledge has influenced on what mathematics contents are used to structure children's play (Kamii & Ewing, 1996).

Vygotsky (1978), on the other hand, believed that children learn through their social and cultural interactions. He states, therefore, that children cannot be understood apart from these social and cultural settings. While Piaget's views on cognitive development have shaped what mathematics contents are deemed appropriate for young children in each development stage, Vygotsky's emphasis on social contexts have influenced how educators encourage young children's mathematical knowledge and ability through appropriate classroom interaction and educational contexts of experience (Bussi & Bartolini, 1998). In Vygotsky's theory, social interaction is a major factor in children's cognitive development, since individual children construct their understanding through interaction with peers and adults who are more knowledgeable (Worthington & Carruthers, 2003). Vygotsky's theoretical contributions to the development of curriculum and pedagogy indicate that the quality of effective mathematics instruction is crucial in terms of the effect it produces on cognitive development (Moll, 1990).

Vygotsky also thought that play is always symbolic, purposeful and rule-bound (Frost, Wortham, Reifel, 2008). For example, games have rules, which reflect social and cultural contexts. Vygotsky (1978) agrees that there are different types of play in which young children engage, but play for him is more social than in Piaget's theory. For Vygotsky, it is the prime arena for children to learn. He believed learning happening first socially and then learning is internalized by the child. He disagreed that the rules for play develop later and noted that young children must be able to follow the rules of society, in order to pretend and take on pretend roles. From Vygotsky's perspective, learning and teaching mathematics should be highly social and mediated by one's culture (Bransford et al., 2005; File, 1995). Robust knowledge and understandings of mathematics must be socially constructed through talk, activity, and interaction around meaningful problems and tools (National Research Council, 2000).

UNDERSTANDING OF A TEACHER'S CONSTRUCTION OF KNOWLEDGE AND PRACTICE

In the history of U.S. early childhood teacher education, constructivism has been continuously considered for young children's learning and teaching (Forman, 1993). Given this, the notion of classroom discussions, grounded on language and its function in an individual's knowledge acquisition and mathematical development, is certainly neither new nor innovative to teachers. However, despite their efforts to learn the importance of verbal and social interaction and to recognize the teacher's role in it, many teachers are still confused and anxious about how to initiate and facilitate classroom discussions (Lee & Ginsburg, 2009). Various factors influence the substantial differences between the teacher's personal understandings and teaching practices in teaching mathematics. The factors include: teacher education and professional development, education policy, colleagues, teachers' life and career stages, and the social contexts of teaching (Floden, 2001).

Clark and Perterson (1986) illustrate these variances that affect a teacher's thinking, planning, and decision-making in the following two domains. As the first one, teachers' thought processes usually occur "inside teachers' heads and thus unobservable" (p. 257) and this domain is deeply intertwined with teacher planning, teachers' interactive thoughts and decisions, and teachers' theories and beliefs. As the second one, teacher's actions and their observable effects are often constrained "by the physical setting or by external influences such as the school, the principal, the community, or the curriculum" (p. 258). Clark and Perterson (1986) points out that there is a reciprocal relationship between these two domains of teacher thought and action.

To understand a teacher's construction of understanding and practices concerning mathematical discussions, I consider how a teacher explains her own thoughts, ideas, assumptions and inquiries about teaching mathematics. I further consider how a teacher makes a decision to act, react, and interact moment-by-moment during mathematics lessons. I also try to have a broadened view of the teacher in context and to look at other "factors from outside the classroom that influences [teacher thought and action]" (Hammerness et al., 2005, p. 378). These include the climate of her kindergarten, school, and school district, parental expectations, the pressures from colleagues (Goldstein, 2007), mandatory standards (Wien, 2002, 2004), and accountability systems (Cimbricz, 2002).

CULTURE, RACE, AND LANGUAGE PROFICIENCY IN MATHEMATICAL DISCUSSION

Student's mathematics learning, from a socio-constructivist viewpoint is highly mediated by social interactions with a teacher and peers in classroom discussions, and it also cannot be understood apart from their real life settings (Bonk & Cunningham, 1998; Erickson, 1996; Wertsch, 1991). Socio-constructivists argue that one of the key premises is referred to as cultural mediation (Ageyev, 2003). A student can come to learn the shard knowledge of their culture, tradition, and language in his/her society (Moje & Lewis, 2007; Mclntyre, et al., 2001), and she/he thus derives meaning from these interactions that affect his/her knowledge construction in mathematics.

Since “mathematics has traditionally been taught from a very narrow perspective” (Mukhopadhyay, Powell, & Frankenstein, 2009, p. 65), many teachers have tended to assume that the principles of mathematics might be universal regardless of language and culture, and that mathematics might be learned and taught in a neutral or objective way. They might believe that desirable mathematics education for social justice and equity is “the same treatment for everyone so that all students have an equal chance to meet the same standards and an equal opportunity to master those standards” (Kahle, 1996, p. 4). However, this basic belief in an equal chance or a fair opportunity is not sufficient to fully incorporate the notion of social justice and equity into mathematics education for all diverse students. This is because any academic discipline, even mathematics, is not neutral (Mukhopadhyay et al., 2009).

Tate (1997) argued that whereas gender differences in the mathematics achievement were small and generally not significant, the gaps “among the scores of students from various races and ethnic groups have slowly narrowed; however, African American and Hispanic students continue to perform at significantly lower levels than White and Asian American students” (p. 673). Similarly, Lubienski (2002) reported that the lower SES and minority students were reluctant to speak about their ideas. They tended to assume that classroom discussions confused them to find the right answers to mathematics problems. In this respect, as the role of culture in student mathematics learning has taken on increased importance, many studies have indicated that the mathematics achievement gaps of cultural, ethnic, and linguistic minority students could be fundamentally derived from a discontinuity between the schools and students' cultural backgrounds (e.g., Gutstein, 2003; Gutstein, 2006). These continued interests have stimulated new models of facilitating linkages between students' experiences in home and school, which can "[empower] students intellectually, socially, emotionally, and politically by using cultural referents to impart knowledge, skills, and attitudes" (Ladson-Billings, 2009, p. 20).

The growing attention to the discontinuity between these students' family and schools' experiences also have led to concerns about how teachers help students effectively manage the inevitable differences between home and school settings (e.g., Frankenstein, 1990). In particular, teachers need to rethink reflectively about the role of their own language in classroom discussions. Khisty (1995) views the communication process in teaching and learning mathematics as "socially contextualized instruction" (p. 295). She argues that effective mathematics instruction for diverse students must be based on "making bridges between meaning and terminologies developed in the two contexts, home and school" (p. 282). Teachers must be alert to how their clarity of wording and choice of language in classroom discussions can be highly related to the processes of school success and failure of ethnic and linguistic minority students (Edelsky, 2006). Teachers should also have questions about how their own cultural, and linguistic backgrounds can influence their actions and thoughts in the process of interacting and communicating with students.

As briefly illustrated above, I, as a researcher, acknowledge that various factors of both students and teachers' cultural, ethnical, and linguistic backgrounds can affect the classroom discussion as a social context of learning and teaching mathematics. These factors are important to understanding how a teacher and students interact in classroom discussions. Nevertheless, not all these factors are studied here. This is because my primary research focus is on understanding how an experienced teacher represents her conceptions, experiences, and decisions for discussion-intensive mathematics lessons through her own language and actions. In the current study, I directly look at a teacher's instructional strategies to initiate and scaffold student discussions in the path of their mathematics learning.

CHAPTER SUMMARY

This chapter illustrated my theoretical framework to explore the research questions in five aspects. I first outlined the socio-constructivists' language viewpoints as my theoretical lens to look at mathematical discussion. Second, I explained Goffman's (1981) conception of the participation framework as a point of departure to understand how participant roles in discussion contributed to children's learning. Third, I illustrated two primary theoretical frameworks of Piaget and Vygotsky of how to understand and support young children's mathematics learning in classroom discussions. Forth, I explained my conceptual framework for looking at a teacher's understandings and actions within the complexity of her teaching practices. Last, I briefly draw on how a teacher and students' cultural, ethnical, and linguistic backgrounds affect their interactions and communication in the process of mathematical discussion, and I explained why all these factors are not mainly studied in this study. These five components help me explore how my participant teacher explains the pedagogical importance of mathematical discussion for young children, and how she enacts that pedagogy in facilitating mathematical discussion in her kindergarten classroom. The next chapter reviews empirical studies on the relations between student learning, teacher's role, young children, and mathematical discussion.

Chapter III. Literature Review

Following socio-constructivist assumptions on language and social interactions in the learning process, many researchers alike are calling for a focus on classroom discussions as socially mediated contexts for learning mathematics (O'Connor, 1998). Chapter Three reviews in detail empirical studies on mathematical discussion. First, I explore relevant research on how students' participation in mathematical discussion relates to their actual mathematics learning. Second, I discuss a teacher's role in initiating and facilitating mathematical discussion. Last, I describe which instructional strategies have been studied and suggested regarding appropriate mathematical discussion for young children.

STUDENT'S ACTUAL MATHEMATICS LEARNING AND MATHEAMTICAL DISCUSSION

The socially constructed interactions typically mediated through language are embedded in classroom discussion that can play a crucial role in developing students' profound, deep, and thoughtful understanding of mathematics (O'Connor, 1998). When students are challenged to clarify their reasoning about mathematics and to elaborate the results of their thinking to others, they are able to make their mathematical ideas visible and improve their ability to reason logically (Chapin et al., 2003). Students also can become more motivated through listening to others' explanations about their reasoning, debating alternative approaches to problems, and sharing their thoughts and methods with each other (Yackel & Cobb, 1996). Moreover, arguing and defending their own positions, receiving and providing a critique of math ideas, and reflecting on their own thinking processes in discussion can allow students to add to, modify, and rebuild their own problem-solving strategies (O'Connell & O'Connor, 2007). Discussions further enable students to display and justify their math ideas, so that they can learn to be clear and convincing and develop their own understandings of mathematics. These processes of classroom discussion thus offer necessary opportunities for students that challenge, stimulate, and extend their own mathematical thinking and reasoning. The expression of student ideas also provides key resources for teachers that inform them about what students already know and what they need to learn (Walshaw & Anthony, 2008).

The positive effects of providing repeated opportunities for students to engage in mathematical discussion have been well documented (e.g., Lampert, 1990; O'Connor, 2001; White, 2003). For instance, O'Connor and Michaels (1996) highlight that students' mathematical argumentation can be shaped by fostering their involvement in taking and defending a particular position against the claims of other students. Furthermore, repeated participation in mathematical discussion can be highly beneficial for even low-achieving students. Baxter, Woodward, and Olson (2001) point out that whereas high-ability students tend to actively engage in classroom discussions, low academic achievers typically remain passive. That is, although low achievers seem to physically get involved in discussion, they rarely express their ideas; while their peers are speaking, they are easily distracted. Similarly, Lubienski (2002) reported that the lower SES and minority students were reluctant to contribute to mathematical discussion. She also found that these students talked about their role in discussion as obtaining correct answers to specific problems, and they stated that different mathematical ideas in discussion could make them confused to find right answers. Empson (2003) argues that the nature of success and failure in mathematics "depends fundamentally on the teacher's role in making space and meaning for students' contributions to classroom discourse" (p. 306). She found that two low-achievement students' participation in classroom discourse about fractions was influenced by the interactions with their teacher, and that the consequences of these students' active engagements could lead to the development of their competencies and identities in learning mathematics (Empson, 2003).

TEACHER'S ROLE IN MATHEMATICAL DISCUSSION

As classroom discussion is recognized as a key part of effective mathematics teaching (Nathan & Knuth, 2003), the expected role of the teacher is changing from a "dispenser of knowledge and [an] arbiter of mathematical correctness" to "an engineer of learning environments in which students actively grapple with mathematical problems and construct their own understandings" (Stein et al., 2008, p. 315). As Chapin et al. (2003) point out, the goal of classroom discussion is not to increase the amount of talk in mathematics classrooms, but to increase the amount of high quality talk or mathematically productive talk to develop individual students' own mathematical thinking and reasoning. This instructional process depends on the skillful orchestration of classroom discussion by the teacher. O'Connor and Michaels (1996) illustrate this aspect of the teacher's role in mathematical discussion as follows:

> The teacher must give each child an opportunity to work through the problem under discussion (whether publicly or privately) while simultaneously encouraging each of them to listen to and attend to the solution paths of others, building on each other's thinking. Yet she [or he] must also actively take a role in making certain that the class gets to the necessary goal: perhaps a particular solution or a certain formulation that will lead to the next step. She [or he] may need to make judgments about what to avoid, or to lead them away from topics or methods for which too many of them are not prepared, while not squelching those who made the problematic contribution. Finally, she [or he] must find a way to tie together the different approaches to a solution, taking everyone with her [or him]. At another level—just as important—she [or he] must get them to see themselves and each other as legitimate contributors to the problem at hand. (p. 65)

In the interactive process of mathematical discussion, the teacher needs to play a pivotal role in initiating participation frameworks for children's engagement and articulating their mathematical ideas, thereby developing their actual mathematical understanding. In this light, I here review the pedagogical strategies for students' participation in mathematical discussion that enables students to become robust learners of mathematics.

First, pedagogy that contributes to students' active engagements in mathematical discussion must place emphasis on "building a feeling of community" (O'Connell & O'Connor, 2007, p. 14) that can allow students to feel that their contributions are listened to and valued. When students feel comfortable with learning together, sharing their confusion, and celebrating their insights, they can have opportunities to promote their willingness to participate actively in classroom discussion, to encourage their confidence about their own intellectual ability to engage in intellectual discussion, and to experience cooperative learning situations (MaClain & Cobb, 2001).

The next focus is to provide cognitively demanding tasks that must embody important mathematical ideas, be solved in multiple ways, and create challenges in students' math conceptual understanding (Hiebert & Wearne, 1993). In particular, when these tasks allow students to pose a variety of questions drawn from their everyday life experiences (Amos, 2007), engagement in classroom discussion can offer motivation and excitement to students. Further, as Empson (2003) points out, use of tasks that elicit the students' prior understandings can facilitate their use of informal resources and ideas and generate their new strategies to solve math problems within their "zone of competence and comfort" (DiSessa, 2000, cited in Empson, 2003, p. 337).

Another crucial facet of facilitating mathematical discussion is teacher talk. Through carefully listening to students' talk and attentively noticing significant mathematical moments, teachers must respond appropriately (Sherin, 2002). Relevant and meaningful teacher responses to student talk must involve drawing out specific mathematical ideas and methods (Hiebert et al., 1997). Particularly, as O'Connor and Michaels (1996) suggest, the teacher's use of "revoicing" (p. 71), which means the repeating, rephrasing, or expanding of student talk, can lead students to clarify mathematics content, extend reasoning with new ideas, or move discussion in another direction. Teachers also must appropriately pose open-ended and higher-order questions that enable students to rethink their mathematical ideas deeply, express their own ideas, and employ new problem-solving strategies insightfully (Hiebert & Wearne, 1993). Through these types of questions, which are more dialogic in nature, provide opportunities that enable students to actively engage in the construction of mathematical knowledge (Case & DeFranco, 2002).

To orchestrate mathematical discussion, teachers also must sensitively know when to step in and when to step out of classroom discussion (Lampert & Blunk, 1998). As Walshaw and Anthony (2008) point out, such sensibility about when to intervene is a rich resource for teachers to make differences in students' mathematics learning. According to Rittenhouse (1998), when stepping into discussion to help students acquire competence, teachers as participants listen to students' talk and ask questions about their ideas; when stepping out of discussion to allow all students to comprehend what is going on, teachers as commentators anticipate where the bumps in the conversational road might be and which points in conversation might be necessary to slow down or rewind. Each of these roles plays a part in helping students gain control over classroom discussions and improve their intellectual ownership in learning mathematics (Rittenhouse, 1998). Without this responsive pedagogical support in discussion-intensive classrooms, the desired outcome of students' advancement of mathematical ideas becomes elusive (Walshaw & Anthony, 2008).

YOUNG CHILDREN AND MATHEMATICAL DISCUSSION

In order to initiate and facilitate mathematical discussion in young children's classrooms, early childhood teachers need to use the instructional strategies, illustrated in the previous section, in more developmentally appropriate ways (Copple & Bredekamp, 2009) and in more play-centered approaches (Ginsburg, 2006).

Developmentally appropriate practices and play

As Schwartz and Brown (1995) point out, four- and five-year-old children are "at that stage of language development in which their thinking far outpaces their ability to verbalize" (p. 350). When teachers ask young children to explain their mathematical ideas, they "often respond with simplistic statements that grossly understate the complexity of their intuitive understandings" (Schwartz, 2005, p. 111). In this light, teaching strategies for best practices in engaging young children in mathematical discussion should be developmentally appropriate to children's age, as well as responsive to their understandings they already acquired and the social and cultural contexts (Copple & Bredekamp, 2009; NAEYC & NCTM, 2010).

To create appropriate practices for young children's developmental status, many early childhood experts recommend that teachers must integrate playful approaches into instructional strategies for orchestrating mathematical discussion (e.g. Copple, 2004; Clements, 2004). Play has been a well-established curriculum component in early childhood education (Frost, Wortham, & Reifel, 2008). When children have hands-on experiences manipulating various objects in discussion activities, beyond just using paper and pencils, they can be more motivated to participate in thinking and talking about their math ideas with each other (Schwartz & Brown, 1995). Further, when teachers provide math concepts through fun activities, children can be more stimulated to participate in discussion situations for learning math concepts, beyond what is generally expected for their age (Kamii & Anderson, 2003). Through play as a developmentally appropriate tool, teachers can create the meaningful learning contexts that stimulate young children's motivation and engagement in mathematical discussion, thereby developing their mathematical thinking and reasoning in the early years.

Furthermore, teachers need to consistently incorporate mathematical discussion with young children's informal knowledge and real-world situation (Copple & Bredekamp, 2009). Mathematics is in the everyday lives of young children, and they are exposed daily to various opportunities to learn math everywhere. Before learning mathematics in structured lessons at school, young children can use "informal skills and ideas relating to number, shape, and pattern as they play with blocks or read storybooks" (Ginsburg, 2006, p. 145). When this informal knowledge of young children is involved in play learning situations, when their existing experiences are connected with the school math experiences (Cooke & Buchholz, 2005), the process of mathematical discussion can offer more purposeful opportunities to develop their math understanding effectively within an individual appropriate level. Thus, to make decisions that ensure teachers' practice is developmentally appropriate, pedagogical strategies for mathematical discussion must be responsive to the everyday math experience of each child that is based on the social contexts in which he/she lives.

Increasing emphasis on academic learning and accountability in early years

Increasing emphasis on academic learning and accountability appears to have led to a corresponding decline in the general understanding of the important contribution that high-quality play can make on children's cognitive development in the early years (Frost, Wortham, & Reifel, 2008). It enforces early childhood teachers to feel that play might be ineffective at preparing children for standardized achievement mathematics tests. At the same time, there is growing pressure for academic pursuits. This makes the teachers favor a highly scripted approach or a teacher-directed method focused on rote learning and memorization. They eschew a child-centered approach or a teacher-guided instruction through classroom discussions (Goldstein, 2007; Jung & Reifel, 2011).

Many studies, however, reveal that learning skills unassociated with play can negatively affect young children's mathematics learning (e.g., Elkind, 1987) and the benefits associated with play can influence young children's achievement in mathematics (e.g., Hanline, Milton, & Phelps, 2008). These studies indicate that the teachers should appropriately balance play with academics in early childhood education, thereby encouraging mathematical play in young children's everyday lives (e.g., Ginsburg, 2006; Schwartz, 2005). Walshaw (2009) also points out that effective teachers encourage their young students to talk, express, explain, communicate, and justify their mathematics ideas. Moreover, through mathematical communication with the teacher's guidance, "students become less preoccupied with findings the answers and more with the thinking that leads to the answers" (p. 19).

CHAPTER SUMMARY

In this chapter, by reviewing previous studies in terms of the teacher's role in mathematical discussion, I found that mathematical discussion is the effective tool for mediating young children's mathematical understanding, and that repeated experiences in participating mathematical discussion are a key part of developing young children's mathematical knowledge and skills. Early childhood teachers must appropriately and effectively use pedagogical strategies for mathematical discussion that motivates and scaffolds young children’s learning in meaningful ways. A next step toward instituting these suggestions from this literature review is to illustrate research methodology to investigate how a teacher conceptualizes his/her role in mathematical discussion and how a teacher orchestrates young children’s participation in mathematical discussion in the kindergarten classroom.

Chapter IV. Research Methodology

Chapter Four details this study's research methodology, describing research paradigm and research design, discussing the participant, data collection, data analysis, the procedure for establishing trustworthiness, ethical considerations, and researcher positionality, and concluding with the study timeline.

RESEARCH PARADIGM: INTERPRETIVE APPROACH

This study adopted the interpretivist, rather than the positivist, approach to the data. Interpretivism refers to the philosophical stance lying behind a methodology "in attempts to understand and explain human and social realities." These emerge in contradistinction to the positivist approach (Crotty, 1998, p 66). Positivists assume that an apprehendable reality independently exists in the form of time-free and context-free generalizations (Guba & Lincoln, 1998). They would follow "the methods of the natural sciences and, by way of allegedly value-free, detached observation, seek to identify universal features of humanhood, society, and history that offer explanations and hence control and predictability," (Crotty, 1998, p. 67). For interpretivists, on the contrary, realities are "apprehendable in the form of multiple, intangible mental constructions, socially and experientially based, local and specific in nature, and dependent for their form and content on the individual persons or groups holding the constructions" (Guba & Lincoln, 1998, p. 206). This is because they believe that human beings socially construct realities as they engage with the world they are interpreting (Sipe & Constable, 1996).

The interpretivist approach was suitable to this study's purpose. That purpose was to understand, rather than to discover, one kindergarten teacher's own ways of thinking about and facilitating mathematical discussion that were socially constructed "in and out of interaction between [the teacher] and [him/her] world" (Crotty, 1998, p. 42). Namely, this approach helped me understand how my participant teacher represented her realties, in terms of mathematical discussion, through her own language, and to look at the meanings and motives behind the teacher's actions and interactions with students in her classroom. It also guided how I analyzed and interpreted her conceptions, experiences, and decisions in relation to her creation of mathematical discussion for young children's mathematical understanding. In adopting interpretivism, I therefore focused on describing the complexity of teaching practices by an individual teacher according to "a set of subjective principles peculiar to that person" (Sipe & Constable, 1996, p. 158).

RESEARCH DESIGN: A QUALITATIVE CASE STUDY

To explore one kindergarten teacher's conception and practice of her role in mathematical discussion for young children, this research was designed as a qualitative case study (Merriam, 1998; Stake, 1995; Yin, 2009). As compared to other types of qualitative research, a case study can provide intensive, holistic descriptions and analyses of a single entity, phenomenon, or bounded system (Merriam, 1998), as well as help to gain an in-depth understanding of the situation in which a phenomenon is inseparable from its context (Yin, 2009). It was well suitable to investigate the complexity of teaching practices in terms of mathematical discussion, because my research interest was “in process rather than outcomes, in context rather than a specific variable, and in discovery rather than confirmation” (Merriam, 1998, p. 19). Because a case study was used, I was able to investigate the teacher’s ideas and practices in detail. I could see, for example, the words and phrases she used, over a number of days and weeks, to begin and facilitate classroom discussions.

Specifically, this exploration was designed as an intrinsic case study by investigating one participant as a single unit. An intrinsic case study is defined as the design “if the study is undertaken because, first and last, one wants better understanding of this particular case” (Stake, 2005, p. 445). That is, the researcher employs this method if his/her intrinsic interest is in the case itself, rather than if the case represents other cases. This design is highly recommended when the researcher attempts “to learn about a little-known phenomenon by studying a single case in depth” (Johnson & Christensen, 2007, p. 408), rather than to provide a general understanding of a phenomenon using a particular case or to generate the findings of a case study to other situations (Yin, 2009; Merriam, 1998). In this light, I used a single-case design rather than a multiple-case design, because my research focus was to gain an in-depth understanding of one particular experienced teacher’s decision-making process for creating and facilitating discussion-intensive mathematics lesson. The focus included the specific situation of her school and classroom she in which she was involved. Accordingly, an intrinsic case study with a single-case design was an ideal qualitative method for investigating an individual teacher’s way of knowing and thinking about her role in mathematical discussion within her own classroom, as well as for learning her professional strategies to orchestrate mathematical discussion for young children within today's public school systems.

Through the use of rigorous qualitative case study research, the focus of this study was on investigating the following two questions:

1. How does a kindergarten teacher conceptualize his/her role in mathematical discussion for young children?
2. How does a kindergarten teacher orchestrate young children's participation in mathematical discussion during mathematics lessons?

SUBJECT SELECTION

This study employed a single-case design with one participant. To select the most promising participant to be studied, I utilized purposeful sampling. According to Patton (1990, 2002), purposeful sampling is well known and widely used as a non-random method of sampling where the researcher selects "information-rich cases for study in depth" (p. 169). It is based on the assumption that the researcher wants "to discover, understand, and gain insight and therefore must select a sample from which the most can be learned" (Merriam, 1998, p. 61). Purposeful sampling was thus suitable to an in-depth focus on understanding how one carefully selected kindergarten teacher conceptualized his/her roles and implemented instructional strategies to orchestrate mathematical discussion with children, rather than by gathering standardized information from a large, statistically significant sample.

To begin purposeful sampling (Patton, 1990, 2002), I determined what criteria guide the selection of a good case for the purpose of this case study. First, the participant should be an experienced teacher who has five years or more teaching experience in a public kindergarten (O'Connor, Fish, & Yasik, 2004), in order to ensure that he/she fully understood the mathematics contents to be taught for kindergarteners and the required curriculum, as well as being familiar with the contextual factors of his/her school systems. The participant also needed to meet additional selection factors for a good case (Stake, 1995) including a recommendation from an administrator and his/her involvement in professional development activities.

I next proceeded to find and recruit a participant matching the criteria I established for my purposeful sampling. As the first step to find potential participants, I sent, throughout a mid-sized city in Texas, emails to several school district directors in charge of external research requests. I asked that my study be conducted in their districts. I also tried to find potential participants through my existing relationships with teachers and principals in one of those districts.

After several months, I had found five potential participants, who had six to thirty years of teaching experience in kindergarten and were willing to voluntarily take part in my study. Then I sent an email to them and explained my research purpose and the expectations of the participant. I also asked them to permit one-day observation before selecting the final participant. I tried to set up a schedule so as to observe, within a week, each teacher's mathematics lesson. This is because these potential participants worked together at the Arbor Elementary School[1] (AES), located in the northern part of a mid-sized city in Texas. Together they developed kindergarten curriculum and mathematics lessons, as well as sharing the same materials. These aspects enabled me, by comparing their teaching practices, to focus more on each participant's teaching style, instructional strategies, and interactions with students.

Choosing Ms. Kelly

After observing five potential participant's mathematics lessons in the third week of November 2010, I decided, for several reasons, to conduct this dissertation study with Ms. Kelly. The most critical rationale was that although five kindergarten teachers implemented the same mathematics lesson plans and used the same teaching materials, Ms. Kelly's classroom, as compared with other four teachers (Ms. A, Ms. B, Ms. C, and Ms. D), seemed to be more filled with the most "mathematically productive talk[s]" (Chapin et al., 2003, p. 6) between student-to-student and student-to-teacher. First of all, unfortunately, there was a small amount of talks in Ms. A's classroom. This teacher directly introduced what the students should learn and do at that day. She did not try to encourage students' motivations using storybooks or small activities; she asked the students to sit at their tables and distributed mathematics worksheets. After the students finished them, she did not provide students with chances to share their work or results with others. They simply got ready for lunch. Next, there was a certain amount of talks in the other three teachers' (Ms. B, Ms. C, and Ms. D) mathematics lessons. They used both

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[1] All names, including those of the schools, participants, and students in this study, are pseudonyms.

small- and large-group of activities to provide students with opportunities to voice their ideas. As students worked, they were busy circulating throughout the room figuring out what ideas they had or what problems students had. Their classroom dialogues, however, seemed not to support student learning intellectually. On the other hand, my observations of Ms. Kelly's mathematics lesson demonstrated that she was outstandingly suitable for Chapin et al.'s (2003) viewpoint of the mathematically productive talk. That is, Ms. Kelly attentively listened to students' ideas; appropriately posed questions to them in order to clarify, add, and make more obvious some problematic features they had; and appropriately allowed them to consider her questions for quite some time without providing them with answers. Through these efforts, she tried to carefully connect their talks and thoughts with the content of the mathematics lesson on that day. In contrast to the other four teachers, I also felt that she tried to react sensitively to her students' emotions to create more comfortable classroom climates (Meyer & Turner, 2007).

Another reason was based on the principal's recommendations. When I met with the principal to submit the papers about my research purpose and to illustrate criteria for selecting the participant, the principal highly recommended Ms. Kelly. The principal, who had worked with Ms. Kelly for more than 10 years, spoke with conviction that Ms. Kelly was the most excellent teacher in terms not of only mathematics lessons but of attitude to students, the management of classroom, the communication with parents, and the collaboration with colleagues. Additionally, two university facilitators who had worked in this school for four years confirmed that Ms. Kelly was the best teacher of her kindergarten team. These facilitators were there regularly to support the professional development of preservice teachers and consistently observed all the teachers' mathematics lessons in this school's kindergarten. They understood the strengths and weaknesses of each teacher's teaching practices as well as the school context. Both of them recommended Ms. Kelly as one who met certain criteria of my research. Based on my observation on her mathematics lesson and personal recommendations from her principal and two facilitators, I finally decided to purposefully select Ms. Kelly as the most promising subject to best answer my research questions (Merriam, 1998). Before starting the study, the selected participant and her students' parents were notified of their rights and the process of the study, and I obtained their written consent to participate.

The participant, Ms. Kelly, is a kindergarten teacher working at the Arbor Elementary School. In her mid-50s, she is a self-identified Caucasian woman, who speaks English as her first language. The year this study took place, she had in total over 30 years of teaching experience. She taught third grade for 5 years, first grade for 1 year, and kindergarten for 28 years. She grew up in Florida and there received her bachelor's degree and earned her early childhood certification. When she moved to Texas, she had to take more courses for her Texas certification. She later received a Masters degree in curriculum and instruction. She taught one year in Florida and then thirty-three years at AES. Several years previous, she had received an award for Teacher of the Year, which was voted on by parents.

Description of Ms. Kelly's school and class

This study was conducted at Arbor Elementary School (AES), opened its doors in 1978. AES is included in the Springfield Independent School District (SISD), located in the northern part of a mid-sized city in Texas. According to the campus profile in the state's 2010-2011 Academic Excellence Indicator System report[2] (Texas Education Agency, n.d.) roughly over 600 students from Kindergarten to 5th grade formed the school body during this period. The student population was approximately 55% white, 24% Asian, 14% Hispanic, 3% African American, 0.2% Pacific, 4% two or more races. Further, 15% of the AES student population was identified as economically disadvantaged, and 9% of the population was identified as having limited English proficiency. AES received an "exemplary" accountability rating[3] for the 2010-2011 school year and earned high performance acknowledgement[4] in reading, writing, science, and mathematics. AES's full day kindergarten program consisted of five classrooms and served approximately 100 children.

During the year that this study took place, Ms. Kelly was teaching 21 students—11 girls and 10 boys. The ethnic composition of the class was as follows: 15 Caucasian students, 4 Asian students, 1 Latino student, and 1 African American student. None of her students were classified as special-needs; there were three ELL (English Language

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2 All numbers and percentages are approximated to protect the school's identity and maintain confidentiality.

3 An "exemplary" accountability rating means a passing rate of 90% of the 3rd, 4th, and 5th-grade students for each subject (Texas Education Agency, n.d.a)

4 The Gold Performance Acknowledgment (GPA) system acknowledges districts and campuses for high performance on indicators other than those used to determine accountability ratings. (Texas Education Agency, n.d.a)

Learner) students and two students with potentially undiagnosed ADHD (Attention Deficit Hyperactivity Disorder) child's behaviors.

DATA COLLECTION

This case study was designed to bring out the details from the viewpoint of the participant by using multiple sources of data gathering techniques of qualitative research (Merriam, 1998) to explore a single case in depth. Three primary types of data were collected from Ms. Kelly and her classroom, including (1) classroom observations, (2) interviews, and (3) documents. First, I observed Ms. Kelly's classroom for 14 weeks. I was careful to observe specifically during mathematics lessons, usually 2-3 lessons each week. The observations included whole-class and small-group discussions. Second, I conducted semi-structured interviews and informal conversations with Ms. Kelly about her mathematical teaching practices. Third, I collected documents such as state/school district learning standards, the teacher's lesson plans, and students' work. These helped me analyze and interpret Ms. Kelly's thoughts and actions of mathematical discussions. I made a chart showing a detailed schedule of the data collection process in Appendix A.

Data were collected over a period of 14 weeks between February 2011 and May 2011. First, the total estimated time for classroom observations was 33 hours. Each classroom observation lasted about one hour and was conducted 2-3 times each week. I visited Ms. Kelly's classroom a total of 33 times. Second, the total estimated time for interviews was approximately five hours. Each semi-structured interview lasted about 30 minutes and was conducted once every two weeks, and I interviewed her six times in all.

I also spent about 3 hours in various informal conversations, one initial interview, and one follow-up interview. The total time for this study was approximately 36 hours.

Classroom observations

As the first major way for collecting data in this case study, I conducted classroom observations. Observational data recorded with field notes and audiotapes offered a firsthand account of the situation (Stake, 1995). This was to aid in understanding the realities of a particular teacher's mathematics classroom, as it exists, without stimulating, manipulating, or imposing any specific event to create a discussion situation (Merriam, 1998; Stake, 1995).

As part of the 14 weeks of data collection, I observed Ms. Kelly's mathematics lessons 2-3 times a week for an hour each time. While collecting information through classroom observation, I acted primarily as an observer not a participant (Merriam, 1998). I took up a position in an unobtrusive location in the classroom such as the back corner of the room, and quietly took notes on a laptop computer focusing on the teacher's verbal and nonverbal instructions and interactions during mathematics lessons.

As the primary tool of recording classroom observations, the participant teacher's mathematics lessons were audio-recorded and transcribed. I set two small audio recorder microphones on a part of the teacher's body and in a certain spot of the classroom in which it could be easy to record all voices of the teacher and students. For closely observing and audio-recording conversational exchanges between small groups and the teacher or between students during parts of mathematics lessons, I sometimes moved an audio-recorder into the proximity of the small-group mathematics activity.

At the same time, classroom observations were recorded in field notes and reflexive journals in as much detail as possible to form the database for analysis. Field notes included descriptions, direct quotations, and observer comments. Once each observation was completed, I recorded field notes as soon as possible after observing. I also wrote reflective journals (Erlandson, Harris, Skipper, & Allen, 1993) as my analytic memos, involving my thoughts, feelings, reactions, hunches, questions, and problems, as well as patterns I had observed, initial interpretations, preliminary analyses, and potential themes in terms of research questions. The researcher's reflexive journals were presented as evidence of an audit trail. Audiotape recording of the participant's mathematics lessons also supplemented my own typed notes in the field notes and reflexive journals.

Interviews

Interviewing is necessary when the researcher want to get meaningful data and to explore the participant's unobservable thoughts, perceptions, feelings, or experiences (Merriam, 1998). When combined with observation and document analysis, it also allows for a holistic interpretation of the phenomenon, in this case, mathematical discussions, being investigated (Stake, 1995). In this respect, interviewing was another important source of this qualitative case study information. Particularly, the semi-structured interview format was ideally suited to ask a participant teacher to describe her understanding, thinking, or experience in terms of mathematical discussion. This is because it is “guided by a set of questions and issues to be explored, but neither the exact wording nor the order of questions is determined ahead of time” (Merriam, 1998, p. 74).

Semi-structured interviews and informal conversations

The research design included six semi-structured interviews and various informal conversations with Ms. Kelly during the 14-week data collection period. She was asked a standard set of questions created for this study (see Appendix B. Standard Interview Protocol). The questions focused on role of a teacher in kindergarten mathematical discussion, including (1) conceptions of the teacher’s role in mathematical discussion, and (2) the teacher’s instructional strategies for mathematical discussion.

Ms. Kelly also was asked prompt questions during semi-structured interviews. These were “in the form of asking for more details, for clarification, for examples” (Merriam, 1998, p. 80) For example, “You said that that activity embeds social studies or social skills in with the math. Am I getting that right? Would you clarify social studies or social skills you talked about?” She was asked follow-up questions or probes developed from my daily observations in her mathematics classroom For example, “When you asked students to make and tell their own addition story related to each math problem, what special purpose did you have for it?; In the today’s small group activity, I observed you divided students into two groups. Why do you think it is important?” These kinds of questions yielded additional information about the participant’s challenges, the relationship between her conceptions and teaching practices, or the hidden meanings behind her decision-making.

Each interview ranged from 30 to 40 minutes. As a non-native English speaker, I intended to avoid any obstacles involving English during the interviews. I decided that, instead of having one or two long interviews, I would have multiple shorter interviews. It was better for me as a researcher to go back and read and make sure I understood everything about the interview and then prepare the next interview. Interviews were audio-recorded and transcribed for analysis. During the process of transcription, pseudonyms were used for the participant's name, students, and school.

Informal conversations with Ms. Kelly often happened before or after mathematics lessons. These informal conversations provided multiple opportunities to test my emerging interpretations of her practices or to ask specific questions about the day I observed. These conversations were recorded in my fieldnotes not audio-recorded. In addition, after finishing all observations and interviews, one follow-up interview was conducted to get additional information about research questions.

Documents

For the third major source of data in this case study, I collected various documents including public records, personal papers, and physical materials. Documents are relatively nonreactive and grounded in the context under study, in that, most are produced independently of the research study, although some documents such as a respondent keeping a diary or writing a life history might be prepared at the investigator's request (Merriam, 1998). In this sense, in order to enhance validity through one of multiple sources to confirm the emerging findings derived from observations and interviews, I collected general information about school profiles, school demographics, state learning standards called the Texas Essential Skills and Knowledge (TEKS), school district kindergarten guidelines and mathematics curriculum. I also gathered the teacher's mathematics lesson plans, the scanned copies of written texts from the mathematics classroom (e.g., students' math journals or worksheets), and the photos of students' works (e.g., what students manipulated using Unifix cubes or what they shaped using Geo-boards and rubber bands). These types of documents helped me uncover meaning, develop understanding, and discover insights relevant to the research problems (Yin, 2009).

DATA ANALYSIS

The analysis of the data in this qualitative case study followed a process grounded in the constant-comparative method (Merriam, 1998; Glaser and Strauss, 1967). This data analysis strategy provided me with a step-by-step way of how to construct meaning from qualitative data.

As the first step, each data set was analyzed individually using the same processes. I began by simultaneously analyzing data after completing the first data collection through observing, interviewing, or documenting (Corbin & Strauss, 2008; Merriam, 1998). For Merriam (1998), as qualitative research is a holistic process, a rich and meaningful analysis of the data should be a simultaneous activity that is located in the interactive process of data collection, analysis, and even reporting. I coded and categorized data through carefully reading and rereading the first set of field notes, the first interview transcript, and the first documents collected, until “any meaningful or potentially meaningful segment of data” (Merriam, 1998, p. 179) was distinguished. As I read and reread each data set, I wrote down emerging comments, thoughts, and questions in the margins, “next to bits of data that strike [me] as interesting, potentially relevant, or important to [this] study” (Merriam, 1998, p.181). Then I checked each unit of data, revisited my marginal notes, tried to group those comments and notes that seemed to go together, and attempted to construct preliminary categories or subcategories (Merriam, 1998). This sequential approach to data collection and analysis enabled me “to identify relevant concepts, follow through on subsequent questions, and listen and observe in more sensitive ways” (Corbin & Strauss, 2008, p. 57) to understand the participant’s role in mathematical discussion. The next set of data was analyzed in exactly the same way as the analytical process of the first data set (Merriam, 1998).

The second step was to construct categories. The preliminary analysis of each data set was periodically reexamined on a weekly and monthly basis (see Figure 1). I began “with a particular incident from an interview, a field notes, or a document, and [compare] it with another incident in the same set of data or in another set” (Merriam, 1998, p. 159). Through these comparisons, I coded data and constructed tentative categories. In alignment with the research questions, the codes were: (a) creating a respectful atmosphere for promoting participants’ willingness, (b) motivating emotions in classroom discussion, (c) laying down ground rules for becoming better speakers and better listeners, (d) respectful talk for managing face-to-face interactions, (e) equitable participation in classroom discussion, (f) purposefully planning discussion-intensive mathematics lessons, (g) scaffolding children's talk to promote their mathematical thinking, (h) maintaining the balance between flexibility and inflexibility, (i) the duality of a teacher's beliefs of discussion depending on mathematics content, (j) the limitation of a teacher's knowledge of content and students, (k) a tight daily kindergarten schedule within mandatory standards, (l) parental expectations induced by pressures of high-stakes standardized testing.

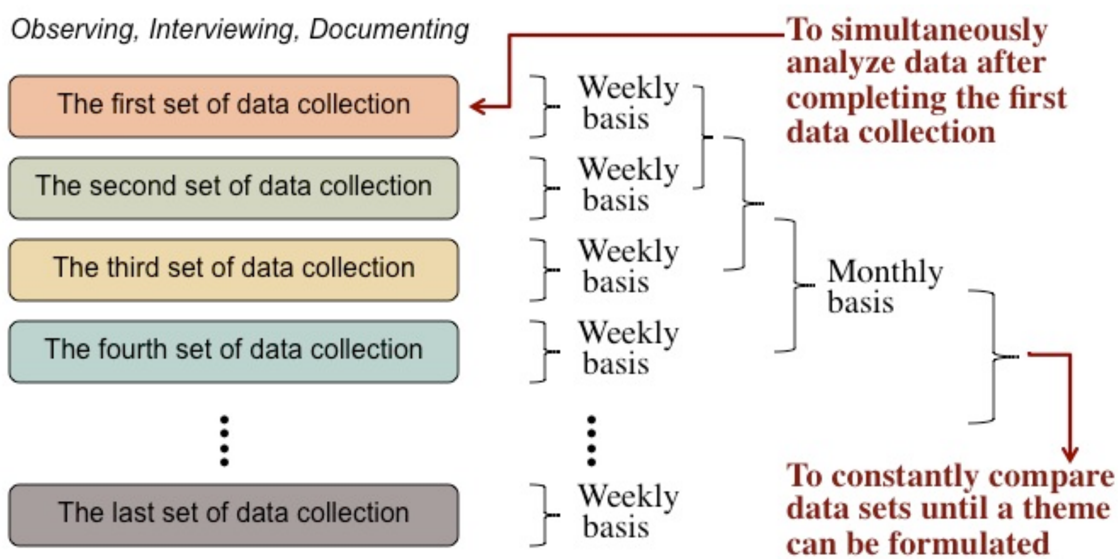

Figure 1. Procedure of data analysis

The third step was to compare data sets in multiple ways to find emerging themes. I compared these categories to each other and to other instances within and between weekly and monthly data sets. I constantly made comparisons "within and between levels of conceptualization until a theme can be formulated" (Merriam, 1998, p, 159). After constantly comparing these coded categories and subcategories, I generated three major themes in terms of this study's research questions, including (1) creating a respectful learning environment, (2) scaffolding student discussions, and (3) overcoming challenges to mathematical discussions.

After completing my analysis of all data sources from the participant, I identified disconfirming evidence, look for data that contradicted statements already coded, seek overlooked information, and make adjustments accordingly in order to concrete my interpretation and conclude my analysis (Merriam, 1998; Yin, 2009).

PROCEDURE FOR ESTABLISHING TRUSTWORTHINESS

To strengthen the trustworthiness of this qualitative case study, several strategies were used in accordance with Merriam (1998): including triangulation, member checking, peer examination, prolonged engagement, the investigator's position, audit trail, and rich, thick description.

In order to enhance internal validity, which was "the extent to which research findings are congruent with reality" (Merriam, 1998, p. 218), I first used triangulation through multiple data sources with class observations, semi-structured interviews, and documents throughout the whole study period. For it, through interviews and informal conversations, I asked the participant of questions to validate what I observed and recorded; I observed focusing on what I gathered from interviews to confirm that she really implemented what she explained during interviews.

Second, for accuracy and clarification of the data constantly, I conducted member checking. As a first level member check, I asked the participant teacher observed and interviewed to review class transcripts, interview transcripts, and field notes, and then to confirm identify instances in which I might have misrepresented or misinterpreted situation, during informal conversations or after interviews. While checking the data, there were some points that she wanted to add or revise, but she said that most of my initial finding and interpretations was accurate. At the end of the study, she also stated, “I gained a great deal out of the probably because you gave me time to reflect on what I did all the time… This was really good for me to sort of reflect over what I've been doing and it helped reinforce some good things that I’ve been doing and so you’ve been very positive” (Interview transcript, 05/19/11).

Third, as a peer examination, I also asked two colleagues, who had doctoral degree in education, to comment on my emerging findings through the perspective of peers. I regularly met them, explained and discussed my analyses and interpretations. Fourth, in order to repeatedly observe the same phenomena, I stayed on-site over a period of 14 weeks until I felt I reached data “saturation” (Corbin & Strauss, 2008, p. 143). Additionally, after establishing preliminary categories and themes, the process of adjusting for disconfirming evidence further enhanced trustworthiness (Merriam, 1998; Mertens, 2005).

At the same time, I intensified reliability, which was “the extent to which there is consistency in the findings” (Merriam, 1998, p. 218), through the following two ways. I clarified my worldview and biases at the onset of the study, as well as explaining in detail the assumptions and theory underlying this study (Merriam, 1998). In order for an audit to take place, I also continuously recorded reflexive journals by describing in detail how the study was conducted and how the findings were derived from the data (Merriam, 1998).

Furthermore, as one of the ways to establish external validity, which was "the extent to which the findings of a qualitative study can be generalized to other situations" (Merriam, 1998, p. 218), I tried to provide rich, thick descriptions so that readers could "determine how closely their situations match[ed] the research situations, and hence, whether findings [could] be transferred" (Merriam, 1998, p. 211).

ETHICAL CONSIDERATIONS

To minimize and avoid potential ethical problems that might emerge with regard to the data collection and in the dissemination of findings in this study, I first notified the participant of the research purpose and the process of the study. The participant was informed of her right that she as a volunteer had control over how much she shared during the observations and interviews. I obtained her written consent to participate before starting to conduct this study. Particularly, because the data of this study included the actions and voices of young children under seven years old, I also obtained parental consent for their children's participation in this study. Before deciding whether or not to take part, the parents received the information about the purpose of this study and its method. They had the right that their participation was entirely voluntary so that they could refuse to participate or stop participating at any time without penalty or loss of benefits.

To protect the privacy of the participant, observation and interview transcriptions, field notes, and reflexive journals were coded so that no personally identifying information was visible on them; all names, including those of the participant, students, and school, were pseudonyms. To ensure the confidentiality of the research data, the transcription and analysis of data collected through the audio-recording were performed at the researcher's home; all collected data were kept securely in a locked box at her residence and were not be disclosed to others. All participants in this study were informed that observations and interviews were being audio-recorded, and this was explicitly stated on the consent form. Upon completion of this study, the recordings from all data were retained to make possible future analysis. Authorized persons from the University of Texas at Austin and members of the Institutional Review Board had the legal right to review these research records and protected the confidentiality of those records to the extent permitted by law.

RESEARCHER POSITIONALITY

As Mertens (2005) argues, "the researcher is the instrument" (p. 247). This suggests that for collecting and analyzing data in qualitative research, validity relies not only on the skill, competence, and rigor of the researcher doing fieldwork (Patton, 1990), but also the social, cultural, historical, educational, and economic contexts that the researcher is experiencing in life (Banks, 1998). That is, any researcher has a certain positionality based on any number of factors. Conducting research within or outside one's own culture determines the positionality of the researcher as "either an insider or an outsider" (Merriam et al., 2001, p. 405), each having certain advantages and disadvantages. In this respect, I clarified my positionality as a researcher at the onset of the study in order to strengthen the trustworthiness of this qualitative case study.

My positionality as a researcher was influenced by the combination of many factors that I experienced in Korea. I approach this study with my teaching experiences as a kindergarten teacher for three years, a part-time lecturer in universities for two years, and my work as a graduate research assistant in the Department of Early Childhood Education for two years. Through observing many early childhood classrooms, I recognized the important role of language in the dynamic process of teaching and learning. While interacting with young children, I reaffirmed that classroom discussion during mathematics lessons is what I regard as the key factor in developing even young children's profound, deep, and thoughtful thinking and reasoning. These experiences greatly shaped not only my beliefs and passions as an educator but also my interests in this research topic.

At the same time, just as Banks (1998) points out, my biographical journey as a researcher had an impact on my values, my research questions, and the knowledge I constructed. I grew up and lived in Korea before I came to Texas four years ago. My personal and academic assumptions and preconceptions thus naturally originated from the cultural, social, and historical contexts of Korea. Banks (1998) defines this positionality as "the external-outsider," who is "socialized within a community different from the one in which he/she is doing research" (p. 8). The researcher with this type of positionality may have "a partial understanding of and little appreciation for the values, perspectives, and knowledge of the community he/she is studying and consequently often misunderstands and misinterprets the behaviors within the studied community" (Banks, 1998, p. 8). On the other hand, Merriam and colleagues (2001) point out that any positionality, either the insider or the outsider, has both strengths and weaknesses in doing research; the outsider's weaknesses can conversely become the strengths. This meant that I, as an external-outsider researcher, had several advantages pertaining to "curiosity with the unfamiliar, the ability to ask taboo questions, and being seen as non-aligned with subgroups thus often getting more information" (Merriam et al., 2001, p. 411). Accordingly, in order to account for the weakness in my positionality as an external-outsider, I was cautious with how much I accurately understood and interpreted my participant's perspectives (Merriam et al., 2001). For this purpose, I was using various strategies such as triangulation, prolonging engagement, a first level member check, a peer examination, and a rich, thick description to establish trustworthiness in this study. On the other hand, my positionality as an external-outsider allowed me to take advantage of the strength in seeing "things not evident to insiders" and rendering "a more objective portrayal of the reality" (Merriam et al., 2001, p. 414) in my study on the complexity of a participant's teaching practice.

CHAPTER SUMMARY

This chapter described the methodology of the research. I first presented explanations and justifications of the use of qualitative case study and interpretive research paradigm to explore one kindergarten teacher's conception and practice of her role in mathematical discussion. I illustrated how to purposefully select a participant and gave background on the research setting including a participant's school district, school, and kindergarten classroom. I also described the procedure of data collection and data analysis. Data were collected through being a careful observer, conducting effective interviews, and mining data from documents. Data were analyzed by the constant-comparative method. To establish trustworthiness, I utilized triangulation, member checking, peer examination, prolonged engagement, the investigator's position, audit trail, and rich, thick description. I provided the procedure to minimize and avoid potential ethical problems that might emerge with regard to the data collection and in the dissemination of findings. Lastly, I clarified my positionality as a researcher at the onset of the study in order to strengthen the trustworthiness of this qualitative case study. In the next three chapters, I present the findings emerged from the analysis of the data.

Chapter V. Creating a Respectful Learning Environment

Chapter Five details the fundamental steps that emerged from the data that go into creating a respectful learning environment for mathematical discussion. Here, I make a series of arguments, focusing on the following five aspects of an early childhood teacher's pedagogy: (1) *creating a respectful atmosphere for promoting participants' willingness*, (2) *motivating emotions in classroom discussion*, (3) *laying down ground rules for becoming better speakers and better listeners*, (4) *respectful talk for managing face-to-face interactions*, (5) *equitable participation in classroom discussion*. Based on these findings, I will discuss major points related to the research questions, focusing on a participant teacher's conceptions and practices of her role in promoting young children's willingness to engage in mathematical discussion.

RESPECTFUL ATMOSPHERE FOR PROMOTING PARTICIPANTS' WILLINGNESS

> On the first day of observation, when I opened the door to Ms. Kelly's kindergarten classroom at Arbor Elementary School, the first thing I felt seemed like the sound of little birds chirping in peaceful woods. Her twenty-one kindergarten students were in their middle stage of a unit on numbers. The lesson at the focus of that day was to understand a number between zero and twenty-five and to verbalize their understandings of the number with appropriate terms such as more, higher, less, and between. The cards with numbers unseen were placed forming a big circle, and her students were walking around the circle and singing together a "Lucky Numbers" song. When the song finished, they stopped and picked up one number card. And then Ms. Kelly asked students to talk to their neighbors about which numbers they had and to compare with the numbers of others, saying such things as, "I have a number greater than thirteen. I have a number less than your number eighteen. My number is in between fourteen and seventeen. Guess what number I have." In a classroom

> measuring about 1,000 square feet, 21 students spoke simultaneously. However, I felt that their voices were harmonious and calm; I saw a kind of pleasure in their faces. When two or three students gathered in a small group and talked with each other actively, I found that they tried to speak in turn and to listen to what others had to say. While students talked to each other, Ms. Kelly roamed around the groups, monitored the students' engagement with the activity, listened to their discussion, and checked for their understanding. (Reflexive journal, 02/15/11)

This scene was I saw on that first day of classroom observation. When I came in and watched her classroom in which the 5-years kids were actively discussing things, I intuitionally know that there had to be talking going on for learning to go on. Although Ms. Kelly's twenty-one students simultaneously spoke and shared their understanding of numbers in a classroom measuring about 1,000 square feet, I felt that their voices were harmonious and their tones there seemed serious and intrigued. Within this atmosphere of calm and restfulness, I saw a kind of pleasure in their faces, and I felt that they seemed to already know how to wait their turn and listen to other's talk. This strong first impression of her classroom made me be prompted to inquire about how it was possible that Ms. Kelly's mathematics lesson was in the peaceful mood despite kids' constant conversations. This question that came into my mind was solved before long, because Ms. Kelly gave me the key clue to solve it at the first interview.

Ms. Kelly indicated that creating a respectful atmosphere was the first step in getting started with mathematical discussion. She explained.

> Many times the students will get the wrong answers, but I really try to dignify their answers and we've had many talks in the classroom about: we don't laugh at wrong answers because it's making fun of other children. And so if a child does have a wrong answer, I try to quickly see how they came about that wrong answer and so I can help them at that

> time – in the moment – get to the correct answer. So I want to remedy this situation immediately and I try to dignify them because I don't want anybody, nobody should feel embarrassed about their learning. And we are in a learning environment and we want it to be respectful. So, my students can feel trusting and comfortable with the teacher and the other students in a learning situation. And I also believe the respectful learning atmosphere helps young students much easily engage in classroom discussion. That's why we do it. I want them to get the right answer but I want them to get it on their own if possible. And eventually I want the children to have ownership of their learning because if they learned on their own, it's their learning. (Interview, 02/24/11)

Ms. Kelly stated that first and foremost in learning should be the respectful atmosphere to encouraging students' willingness to engage in classroom discussion. When students feel unembarrassed and comfortable about their learning, they can get started participating in mathematical discussion and they are eventually able to develop ownership of their learning. Her statement about this strategy reflects Chapin and colleagues' (2003) point that the teacher needs to establish "a classroom culture in which students listen to one another with respect" (p. 20). O'Connell and O'Connor (2007) also assert that students are willing to explore the content of mathematics deeply if they are comfortable expressing their illogical ideas as well as their accurate answers. In these arguments, the emphasis upon comfortable and respectful learning environment stems from the consensus that emotions consciously and unconsciously play an essential role in the process of learning (Do & Schallert, 2004; Mayer & Turner, 2007; Schallert & Martin, 2003; Pintrich, 2003).

MOTIVATING EMOTIONS IN CLASSROOM DISCUSSION

In certain situations, some types of emotions can motivate students, stimulating deep involvement and persistence in classroom discussion as well as providing them with an enjoyment of learning. Alternatively, emotions can also lead students to experiencing anxiety, disengagement, boredom, and distraction. This aspect of emotion for students' participation in learning was often on display in Ms. Kelly's mathematics lessons. For instance, Ms. Kelly's class had been exploring various kinds of subtraction problems. She wanted her students to learn about the meaning of subtraction and then to create, write and solve subtraction problems. After presenting a flannel board poem called, "The Runaway Cookies," she asked students how to draw a subtraction story from it.

Ms. Kelly:	How many cookies did we have? Everyone?
Students:	Five.
Ms. Kelly:	What happened to all the five cookies, Andrew?
Andrew:	They danced away!
Ms. Kelly:	Andrew said they danced away. Aha! They were out playing and they danced away. So… Can anyone tell me many a math story about this?
Claire:	There were five cookies, and then they started dancing and they danced away, and now there are zero.
Ms. Kelly:	Wow that was a great story. And I bet you the person that owned the cookie jar was very sad when they came back, weren't they? OK, who can tell me in a number sentence? What happened? Noah?
Noah:	Five… minus… five… equals… zero.
Ms. Kelly:	Okay, see if you guys agree. Five minus five equals zero [Ms. Kelly writes Noah's number sentence on a whiteboard as Noah states it]. Thumbs up if you agree… Anthony, well how do you feel about it? Do you agree with that? Thumbs up if you agree. You don't agree?
Anthony:	Oh, I do agree.

Ms. Kelly:	Okay, let's see the thumbs up. Oh, Jacob, is your thumb down? Okay, so you don't agree? Why don't you agree, Jacob?
Jacob:	Well… Because… I think… five… plus… five doesn't make zero…. It makes ten…
Ms. Kelly:	Oh. I'm so glad you said that. Why do you think so?
Jacob:	Um… I think five plus five is ten.
Ms. Kelly:	Oh, you're right. Five plus five is ten. But look at this sign. Is that an adding sign or a minus sign?
Jacob:	That's minus.
Ms. Kelly:	So that means, what do we have to do these?
Students:	Take them out.
Ms. Kelly:	Take them away. Now do you agree with it?
Jacob:	Well…
Ms. Kelly:	OK, let me give you an example. If I have five… Here, I will give you five cookies, okay. So, here. I will want them all back. So you add five and then you … the teacher took away five, and now you have what?
Jacob:	[Pause] Zero?
Ms. Kelly:	Zero. You're right. If that were a plus sign, it would be different. What would be if I had a plus sign there, Jacob?
Jacob:	Ten!
Ms. Kelly:	Mm-hmm. And did that happen in this story? Did we have five cookies and five more came and joined them?
Jacob:	No!
Ms. Kelly:	No. That didn't happen in this story, so Noah was going to put a minus sign. Noah, is it right?
Noah:	Unh-huh. I said five minus five.
Ms. Kelly:	So, this tells what happens in this story. Now, do you agree with it, Jacob?
Jacob:	OK, I agree! (Class transcript, 02/24/11)

After hearing Claire's subtraction story, Ms. Kelly asked Noah, one of students raised his hand, to tell the number sentence of it. She addressed that Noah was a bright children, but he lacked confidence. Although his answer was correct, he talked in a timid voice again this time. For this, after writing on a whiteboard the number sentence stated by Noah, Ms. Kelly moved on to invite all students to agree or disagree with Noah's claim. This kind of strategy is a "talk move," which early childhood teachers readily employ to ask young children to apply their own ideas to someone else's ideas (Chapin et al., 2003, p. 11). Through this way, Ms. Kelly wanted to enable Noah really to see how many peers favored his idea, as well as to encourage Noah to be confident of speaking his own thoughts aloud. In doing so, Ms. Kelly tried to support Noah's emotion in order to encourage him to freely and comfortably participate in a whole-class discussion.

It was further effective for Jacob, a passive student who was hesitant to raise his hands and to think out loud in front of his classmates, to engage in classroom discussion without a mass of emotional burdens (Chapin et al., 2003). Ms. Kelly quickly perceived Jacob shaking his head by leaving his thumb down, and she asked him to talk aloud about why he disagreed with Noah's claim. Particularly, because Ms. Kelly was aware of the crucial role of emotion in learning, she wanted to give a supportive response to Noah's wrong idea, to listen to his reasoning, and then to encourage him to participate, safely, in the discussion. Ms. Kelly did not say to Jacob, "No, that was wrong," when he gave a wrong answer. She was careful to avoid making him feel bad. She also wanted Jacob's disagreement to prompt other students to reevaluate and rethink their own ideas (Chazen & Ball, 1995). By saying, "I'm so glad you said that. Why do you think so," Ms. Kelly made Jacob feel okay and comfortable to engage in the process of figuring out which aspects he misunderstood in this problem. In this way, Noah, who had not solved the problem, was able to make progress in his understanding of the subtraction. Also, Ms. Kelly was able to focus the discussion on the procedure that students used to explore the subtraction problem, rather than asking them questions to elicit a correct answer.

Ms. Kelly knew that just focusing on mathematics cognitively was insufficient to lead students to explore the content of mathematics effectively. She recognized that students' emotional reaction in learning mathematics could interact with their cognitive and motivational processes as they took part in classroom discussions (Do & Schallert, 2004). In this respect, she emphasized her role in scaffolding students' emotional responses (Rosiek & Beghetto, 2009). Indeed, she established a respectful learning atmosphere as the preliminary step to promoting their willingness to engage in mathematical discussion.

GROUND RULES FOR BECOMING BETTER SPEAKERS AND BETTER LISTENERS

Ms. Kelly emphasized that initiating and maintaining ground rules helped create a respectful learning atmosphere for a discussion-rich classroom. Such an atmosphere can emotionally motivate students to become better speakers and better listeners in mathematics discussion. This had been her role since the beginning of the fall semester. She illustrated her instructional goal for her math lessons, as follows:

> I wanted my students to verbalize their ideas, and plus I wanted them to develop listening skills with the others, so I wanted them to know that when someone is speaking, they're supposed to be listening. So it's another social studies type of skill again that we were working on during the math time. And they also need to be better listeners, as well as better speakers, too. So we're working on social skills there, too. (Interview, 04/29/11)

Ms. Kelly believed that classroom norms for facilitating mathematical discussion should include a set of social skills that young children would need when interacting with others.

For example, they should not only speak in front of a group and listen to others' talk, as well as they should take turns, wait, be patient, and be friendly. And yet, based on her long teaching experiences as a kindergarten teacher, she recognized that it was challenging because of the age characters of young children. She explained further:

> I taught many kindergarten students for my thirty-three years. You know, most of young children really like to say their experiences and ideas to me or to anyone who is willing to listen. When they enter the classroom, they start to say to me about what happens at home last night, or what they see on the way to school. And even during lessons, they also like to say something irrelevant to the contents that be taught. But, they're young children, so it is natural that many young children cannot wait to raise their hands to say their ideas, and they may interrupt when others are talking. And many of them don't listen well. So they often don't understand what the previous speaker has just said. So, it's not easy for me to manage classroom discussion with young children during math lesson. So the first important thing for respectful talk is to get started to explain what I expect from students in my math classroom. I think it is important to my young students, who are not familiar with social norms for discussion. And, after introducing my guidelines to students, I try to give them chances to discuss how they think, do, and react during discussion, and to decide classroom norms with students. And I made these efforts from the beginning of the fall semester. It looks like a long-term project that should be continuously kept in accomplish until the end of year. And it is a big challenge to me. (Interview, 04/29/11)

As described above, Ms. Kelly was aware of young children's developmental stage of communicating in the kindergarten classroom, which might be regarded as their first or second experience communicating in a public school context. In this respect, she pointed out that her role in introducing the form of discussion and explaining her expectations could be essential to setting up social norms for young children to participate in mathematical discussion (Chapin et al., 2003; O'Connor &, Michaels, 1996).

Furthermore, the supportive and courteous atmosphere in her mathematics classroom had prevailed throughout the whole year thanks to her continuous efforts.

At the same time, she emphasized inviting even young children to join in the process of setting the ground rules for her discussion-rich classroom (Bruner, 1983; Brandt, 1999). For this, she provided students, after explaining her expectations for setting a discussion-centered classroom, with opportunities to think about, discuss, and decide the rules that they should follow for one year (for example, we all have good ideas; we all listen carefully; we all talk ideas; we all share our ideas; we all wait our turns; we all ask any questions). Her strategy could encourage students' willingness and promote their motivation to more actively engage in mathematical discussion. Furthermore, it could help foster their intellectual ownership of their mathematics learning (Rittenhouse, 1998).

She further mentioned that she often reminded students of the norms that together they had decided on. She did this to saturate the process of classroom discussion with the respectful learning climate. For example, Ms. Kelly had students sit by a partner and then said as follows,

Ms. Kelly:	Now, does it mean that the other partner cannot give you advice?
Students:	No!
Julia:	We can talk!
Ms. Kelly:	Right! They can talk to you – And?
Anthony:	We can listen!
Ms. Kelly:	Right! You can listen their ideas and their thinking because they might have a better idea, or you might have the better idea or you might both have the same idea. (Class transcript, 02/17/11)

Ms. Kelly explained her expectations about how students should talk and listen; she guided the students to become familiar with classroom norms to enable courteous discussion. Particularly after winter break, she pointed out the need to help students remember the norms. She often reminded students, as she did in the dialogue above, of how to discuss with others in her classroom. It shows that success in facilitating mathematical discussion depends not only on the teacher's role in initiating ground norms of discussions but also on her role in maintaining, over the course of a school year, a discussion community for respectful talk.

RESPECTFUL TALK FOR MANAGING FACE-TO-FACE INTERACTIONS

According to Ms. Kelly, young children needed to learn respectful talk in order to use mathematical discussion for their mathematical learning.

> The first important one is how to listen; what are good listening skills? We work on that throughout the curriculum. The second one is how to get them to explain, how to speak to someone clearly, to get them to understand it. The third one is just being polite, being a good listener, speaking politely, dignifying each other's answers. (Interview, 05/19/11)

Ms. Kelly pointed out that, to participate effectively in the classroom discussions, it was necessary that children should develop listening skills with understanding and speaking skills to express their ideas clearly, but it was not sufficient. Children should develop specific skills for being good listeners and good speakers to politely manage face-to-face interactions. She was aware that the primary framework of having students participate in discussions should be based on relational roles between being a polite listener and speaker (O'Connor & Michaels, 1996). In this respect, she first described herself as an initiator of setting up respectful talk. As such, she tried to help children become good speakers and good listeners during classroom discussion:

> I love to listen the ideas that children come up with, as many parts of the whole class discussion or small group discussion, because the child has a great idea. But if they can't verbalize it, they do probably not understand. Classroom discussion from student–to-student and student-to-teacher is very important in their learning because they have to be able to explain or verbalize things in life and to really understand it. So I need to have them to get lots of opportunities to do this. At the same time, when they can explain it, the other children can understand it better than if I tried to explain it. Because hearing it from a peer, it has a great deal of impact. And then if a child listen another child's idea and sees another child doing it, then they are encouraged to do it. So that's why I have the children get many chances to listen carefully what someone explain it to other children. So, I'm working on children becoming good listeners and good speakers, so that helps with that, too. (Interview, 04/07/11)

In Ms. Kelly's view, the teacher needs to set the conditions for every student to have many opportunities to express their ideas and listen to another student' thoughts in classroom discussion. This is because the relationships between the roles of speakers and listeners can enable students to better understand and engage more in the process of exploring mathematics problems (Walshaw & Anthony, 2008). Her assumption that talking and listening are important skills connects with what Cazden (2001) calls the "speaking rights and listening responsibility" (2001, p. 82) of mathematical participation. Each student has a right to participate in discussion by speaking out freely, and along with this right comes an obligation to listen to others respectfully (Chapin et al., 2003). Conversely, each student also has a right to hear others' ideas so as to firm up or expand his/her own ideas. Correspondingly, this obligates a student to speak his/her own thoughts to others so as to clarify the student's own understanding and rebuild his or her problem-solving strategies.

Furthermore, Ms. Kelly emphasized each student's responsibility to treat another student with respect.

> I'm hoping that the students will take the behaviors or acceptable practices of working in a group discussion and take it elsewhere in their lives, on the playground, or in other small group situations, or working out issues in the home living area or in other places, too. It's not just for math. They just need to honor each other and be respectful of each other, and I want them to be able to do that on their own. (Interview, 03/24/11)

Ms. Kelly assumed that students' behaviors and discourses, based on the basic norms for courteous discussions, should be associated with their daily practices and lives (Cobb, Wood, & Yackel, 1993). The positive aspect of rules in respectful classroom discussions helps students develop "one such habit that exemplifies courtesy and brings out proper behaviors and civility towards others" (Miller & Pedro, 2006, p. 294).

However, despite of her efforts to establish the norms of participating roles with respect, she reflected that her classroom discussions did not always succeed to follow to the instructional ways she intended. For example, she pointed out the difficulty of facilitating small group discussions with kindergarteners.

> Small group discussions in kindergarten can be very difficult. So, many times I will have children just work with a partner in a pair because it is easier to work one-on-one. But when you get more than three students or even three and higher it's really hard for children to work together, to accomplish something. So in this lesson I gave them exactly what would happen if I said, "Here take this bag of shapes and sort it according to if it can be folded in half or not." That would be having kids grab it and that. And I chose the student that I thought would be strong enough and fair enough to give each child an opportunity. And I gave other students

> specific roles about how they talk, listen and act with the leader. So, I stress in here a lot about giving everyone a chance to learn in small groups. And so I accomplished it. I thought we did a pretty good job on that lesson by choosing someone that could do that. And then I monitored the group and I saw that there was one group that was quite doing it, so I had to come in and intervene a little bit. (Interview, 03/24/11)

Ms. Kelly confessed that whereas partner talk had a certain benefit of allowing young children easier to work one-on-one, it was not easy that three to higher students would cooperatively work together in small group discussions. For this, she asserted that the teacher should give kindergarteners more concrete directions and instructions during small group discussions, rather than during whole group discussions and partner talks. Specifically, she clearly planned and organized how she initially gave each student a certain role, such as the leaders and members of a group, about how to respectfully talk, listen and act with others. She also concretely clarified for students, according to each student's role in a small group, her expectations about their rights and responsibilities. She believed that this sort of her role in initiating participant roles and responsibilities enabled everyone to have a chance to learn in small group discussions.

The following example shows how she specifically explained it to start a small group activity of finding shapes that were easy or difficult to fold equally:

> Ms. Kelly: Now I am going to divide you into four groups and I'm going to give you some objects. I'm going to give one bag to the leader of the group and their job is to take this out and have a discussion. And you have to decide if this is something that could be easily folded or cut into two equal parts. And you would put the things that are easy to put into two equal parts into one pile and the things that might not go into two equal parts easily in another pile. So a circle we know is easy to fold in half. Let's take a school bus. Let's fold it this way. Is

that two equal parts? Is that divided into halves? Or cut into halves?

Students:	No.
Ms. Kelly:	Well, let's try it the other way.
Students:	No.
Ms. Kelly:	No. Well, let's try a diagonal fold here.
Students:	No.
Ms. Kelly:	So, I put this into a not easy to cut into half... Now, if I call your name, sit in circle. So, you four Claire, Kamala, Ethan, Jessica, come up here, sit in a circle. Would you move in circle? Noah, please join here.
Noah:	Sure.
Students:	[Five students who were called by the teacher move in a circle.]
Ms. Kelly:	Claire, you are going to be the leader at the bag.
Claire:	Okay.
Ms. Kelly:	So you take out one piece and you guys discuss it. But if I were you, I'd hand, like, one to Kamala one time and let her do the folding. And then Noah would get the next one, but just do one at a time and just talk about it. The rest of you guys listen carefully, and you can ask any questions to Noah, or you can say any other ideas to Noah's thought.

(Class transcript, 03/22/11)

Ms. Kelly began this part of the fraction lesson by explaining her detailed expectations of students' roles in small group discussions. She explained that she had divided, randomly, students into groups of five; she had foregone using the original groups of six seated at tables throughout the semester. This small group activity, she believed, was not a high level task and thus grouping at this point could disregard the personality and cognitive level of each student. She further explained that when she chose this type of random grouping, her initiation of participants' roles was important to set up small group activities. She thus first allocated individual students' specific roles, such as the leader and group members, and then she assigned relational rights and responsibilities to students: the leader distributed one shape to each student one time, so that each student would have his/her turn to engage in the activity. The group members were, in turn, to say aloud their thoughts, to listen to the others' thoughts and ideas, and to provide alternative ideas.

After each group sat together on the carpet, Ms. Kelly went over to small groups of four students. At this time, she overheard the following interaction:

Claire:	Please pass it [a triangle] around.
Noah:	Ok. Fold it.
Ethan:	Yeah, that's equal. That's easy to do.
Noah:	That's equal. It's equal.
Claire:	Let's put it by me. So equal goes on this side… And then pass one [a bus] to me. This school bus is not going to be. Let me try this. Ooh… No.
Ethan:	Let me try. Maybe… You know what I can try. I can try a diagonal. I can… diagonal and then… curve this over it. And then… curve this back over and then this here and then this… here. There you go. But that was not easy. We'll put it in here. That's the not hard one… I finally figured out how to do that. But I had to fold it more than one time.
Ms. Kelly:	Did it fold equal parts, the bus?
Ethan:	Kind of, yeah.
Ms. Kelly:	Really?
Ethan:	I can show you. Let's see. I can fold it this way [on the diagonal].
Noah:	No, don't try it. I think… Okay, let me try. Bus… It's a hard one. Let's put that into the not hard…
Jessica:	You're right. It's not easy.
Claire:	Ethan, please put it in there.
Jessica:	That means not easy.
Ms. Kelly:	Okay, Ethan. Are you agreed?
Ethan:	I think… Okay, put that in the hard pile.
Ms. Kelly:	Are you sure? Why do you think Claire and Norah want to put this into a not easy to cut into half?
Ethan:	Because… I can try a diagonal, so I fold it into two parts, slightly equally… but I cannot fold it in half exactly.
Ms. Kelly:	So?
Ethan:	I'll put it here… Hard pile.
Jessica:	Ethan, put it in the hard pile next to me.

Noah:	Claire, you are the leader. It can be next to you. You're the leader.
Claire:	Okay…. So, who is next?
Jessica:	Is it my turn?
Claire:	Yes, it is your turn.
Jessica:	Okay, [folding a square into half] that's easy. So just put it in this [easy to fold] pile. Okay, now it's Kamala's turn. Now it's Kamala's turn.
Noah:	What does she get?
Jessica:	It's a pie plate.
Kamala:	I don't know how to fold that.
Jessica:	How about this way?
Kamala:	[Kamala folded a shape of a pie plate according to Jessica's idea.]
Ms. Kelly:	Oh, does it work that way, Kamala?
Kamala:	Yes, kind of…
Ms. Kelly:	Is it two equal parts?
Kamala:	Yes.
Ms. Kelly:	Is there another way?
Kamala:	No.
Jessica:	We can put it in that [easy] pile.
Ms. Kelly:	Is there another way she [Kamala] could fold it?
Ethan:	Uh, no way.
Ms. Kelly:	Ooh.
Ethan:	So put it in the easy pile. Okay, that was easy. Okay, now it's my turn.
Claire:	It's your turn for the string.
Ethan:	That's easy.
Jessica:	Yes.
Ethan:	Yep, that's easy to fold into two.
Claire:	So put it in this [easy] pile.
Noah:	Okay. That's easy. Put it in there.
Claire:	So put it in that pile. We're done.

(Class transcript, 03/22/11)

As described above, this episode showed how a group of kindergarten students, divided into roles of the leader and group members, engaged in a small group discussion initiated by the teacher. Claire begins the discussion by distributing shapes to other members. Her basic responsibility as the group leader was to have all members speak in turn (e.g., "So, who is next?"; "It's your turn for the string."). Such a role helped prevent any struggles between students and enabled them to listen carefully to a courteous discussion. The group-member responsibility was to take part in the discussion—to speak their ideas and to listen to others' ideas. Ms. Kelly indicated that these speaking rights and listening responsibilities provided individual students with opportunities to elaborate their thoughts by verbalizing them and to rebuild their own problem solving strategies (Chapin et al., 2003). She also asserted that "persistent participation" (Sfard, 2006, p. 166) in this format of small group discussions enabled young students to be familiar with ground rules for respectful talk, as well as to improve their willingness to speak, listen, and share their ideas more actively. Furthermore, as O'Connor and Michaels (1996) pointed out, repeated experiences of this kind in teacher-initiated discussions can help gradually develop students' own roles of engaging in discussion.

In this episode, Ms. Kelly's role was to attentively observe how students' discussion went and to appropriately scaffold students' ideas. For example, students had different ideas on how to fold the shape of the school bus into two equal parts. Claire thought the school bus could not be folded into halves equally. Ethan offered a different opinion. He thought that the school bus might not be easily folded in half but after several trials he could do so. Ms. Kelly took his different view as a starting point to raise a dispute between students. She asked Ethan "Did it fold equal parts, the bus." This question offered students a chance to review their ideas and reexamine the problem. After monitoring these students' discussions, she then removed herself from this discussion and moved to other groups. It indicated that Ms. Kelly was aware of not only when to step in to help students acquire their own problem-solving strategies but also when to step out so they could develop their intellectual ownership of learning mathematics (Rittenhouse, 1998).

EQUITABLE PARTICIPATION IN CLASSROOM DISCUSSION

Ms. Kelly further indicated that a teacher should think carefully about ground rules for encouraging the equitable participation of every student in a mathematics classroom. She explained her goal of mathematical discussion:

> My goal is to make sure that all children as listeners and speakers are engaged in discussions in my math class. So they are able to learn math effectively. Even the more passive ones might be engaging in a level that I can't see. But I need to get them to verbalize, as well, and be able to, um, assert themselves, too. So I try to make that happen. Sometimes I'm not successful; sometimes I'm very good at it; but I keep trying. (Interview, 04/29/11)

Ms. Kelly stated that her instructional goal in teaching mathematics was accomplished by how to get all students, as listeners and speakers, to engage in classroom discussion (Chapin et al., 2003). In particular, she emphasized her role in enabling the more passive students to verbalize their ideas to others. And yet, she reflected that she couldn't easily confirm whether each student said at least one thing in the discussion. For this, she used several strategies to encourage every student to contribute, at least once every few lessons, to the mathematical discussion.

Considering the different levels of students

Her first strategy was to start, for low-performing students[5], with easier mathematics problems. During whole group discussions, I often observed that she began by asking such students easy questions. And then she gave higher-level mathematics questions to the rest of the students. She explained her intention:

> In whole class discussion, many times I often ask the easier questions first, to build the scaffolding. So everyone can understand it. And I call on the students that might be having difficulties, a couple questions into it, to make sure that they are with me, that they are engaged, and that they are learning and to help me identify what they know and what they don't know. And then we also get it a little bit harder and those kids I might lose them for a little while but I need to engage the higher levels, too, in the same thing. So everyone is learning from those lower-level questions and I want to make sure those struggling students have an opportunity to be successful. And so I won't ask them questions that have higher numbers, and I will give that to somebody else. So they're all learning the same process, but some might be using lower numbers and some are using higher numbers. But I want all children to be successful and to build self-confidence. And that's why I do that. (Interview, 03/24/11)

By starting with lower level mathematics questions, Ms. Kelly could help stimulate struggling students to participate in whole group discussions. To call those students' attention to mathematics content, she first used the problems that they could solve independently. She then gave more difficult problem that they might handle with assistance. Her conception of this strategy is a derivation of Vygotsky's (1978) Zone of Proximal Development. She provided mathematics problems appropriate for individual students' actual as well as potential intellectual level. In doing so, she wanted all

[5] Ms. Kelly identified the low-performing students based on her daily observation and assessment, stating "I am teaching and assessing at the same time. I was just writing notes down. And the assessment tells me who need some more practice, who doesn't have it, whom I need to do re-teach" (Interview transcript, 02/24/11). In particularly, she indicated that the very low-performing students were Bela, Matthew, David, and Sarah (Interview transcript, 03/24/11).

students, both low- and high-performing, to have opportunities to be successful at learning mathematics. Capitalizing on such opportunities, students built their self-confidence (Lave & Wenger, 1991).

Assigning turns by calling on students

As another instructional strategy to promote equitable participation in whole group discussions, Ms. Kelly also called on students, including the reticent ones, to voice their ideas.

> I have many different settings in which I teach math or have math experiences. And one is the large group. I am constantly, when I'm calling on people, I try to make sure that those who don't discuss much can discuss things. But I try to clue in on what they can discuss, so I don't call on them to discuss something that I know that they don't understand because I want them to be successful in their discussions. I don't keep tally sheets, but I do oftentimes go by row sometimes or I go, "Oops, I've been calling on that child too much. This child I haven't." So those are things that are always going through my mind. I can't document everything. If I document everything, like who has had a chance… I've taught my student teachers, if they are not already doing that mentally or automatically by themselves, how to draw sticks to make sure that everyone has a fair share. But sometimes not having the same amount of time to explain is not always fair, either, because there are some kids that I know can explain so they don't need to be explaining as much. And I will use them when I want a good explanation. The other kids need more than one or two chances, so I try not to have favorites that I call on all the time, but I keep that in mind. (Interview, 04/29/11)

Assigning turns by calling on students, whether or not they have raised their hands, is a traditional method (Chapin et al., 2003). Yet Ms. Kelly believed it to be effective at ensuring all children participate in a large group discussion. She further asserted that equitable participation did not necessarily mean that every student had the same amount of time, in a mathematics lesson, to voice their ideas. From her viewpoint, while a teacher should require every student to participate at least once by calling on him or her, the teacher should also consider giving the silent and/or passive students one or two extra chances.

Utilizing partner talk for silent or passive students

The third instructional strategy that Ms. Kelly suggested for silent or passive students was the discussion format of "partner talk." In an early stage of a unit on addition, she asked students to find two kinds of objects in the classroom, to bring them back, and then to turn and talk with their share-pair partner about how to add the objects together. Joshua, an active student, and Alexander, a passive student, had the following exchange:

Joshua:	Tell me your story first.
Alexander:	(In a low voice) I have five pencils in this (left) hand… And this (right) hand, I get seven cubes… And put them together… makes eleven!
Joshua:	So, you... five and seven equal? What?
Alexander:	Eleven!
Joshua:	Okay, it's my turn. (Class transcript, 03/29/11)

In this speaking format, Ms. Kelly gave students a very short time, one to five minutes, to think about and tell their partners their addition story. She spoke of this format's potential strengths:

> My intention in this lesson was for the children to experience adding in a concrete way. And I wanted everybody to be successful and I wanted to make sure that everybody had a solid introduction to addition and knew what it was. So having concrete things in quantities that they could handle was one of my goals. Having them speak with pairs and their partners give the children more opportunities to speak, instead of having everyone speak

> just one at a time in a large group. So everyone was able to participate and everybody was able to explain what he or she was doing. (Interview, 04/07/11)

Partner talk differs from small group discussion (Chapin et al., 2003), in that both students, whether or not they like to talk, are obligated to verbalize their ideas. Ms. Kelly pointed out that, compared to large group discussion, pair talk offered, even imposed, more chances to speak.

TEACHER'S ROLE IN ENCOURAGING STUDENTS' PARTICIPATIONS IN MATHEMATICAL DISCUSSION

The primary role of a teacher in getting mathematical discussion started for young children was to establish a respectful learning environment that could motivate their willingness to participate in classroom discussions. I made a series of arguments focusing on the following five sub-themes: (1) *creating respectful atmosphere for promoting participants' willingness,* (2) *motivating emotions in classroom discussion,* (3) *laying down ground rules for becoming better speakers and better listeners,* (4) *respectful talk for managing face-to-face interactions,* (5) *equitable participation in classroom discussion*. These five elements demonstrated the teacher's understanding and use of scaffolding students' emotions in mathematical discussion. They also demonstrate the teacher's use of pedagogical approaches "to influence students' emotional response to specific aspects of the subject matter in a way that promotes student learning" (Rosiek, 2003, p. 402). Mayer and Turner (2007, p. 244) define such "emotional scaffolding" as "temporary but reliable teacher-initiated interactions that support students' positive emotional experiences to achieve a variety of classroom goals." In this respect, Ms. Kelly recognized that classroom discussions involved not only the cognitive process of exploring mathematics content itself but also the emotional process of motivating their engagement in learning (Do & Schallert, 2004). She also showed a variety of instructional strategies to create a supportive and respective atmosphere for young children to talk about their ideas comfortably.

She pointed out that the nature of success and failure in the preliminary step to facilitate mathematical discussion fundamentally depends on how a teacher does two things: the teacher must create a respectful atmosphere that scaffolds students' emotional responses and the teacher must lay down ground rules that make the students polite speakers and listeners. That is, on the one hand, the teacher's emotional scaffolding should be a prerequisite for respectful speaking. On the other hand, the teacher must play a role in eliciting participants' rights and responsibilities and in helping every student be able to contribute to mathematical discussion. In this section, I discuss four main aspects of these findings, comparing and contrasting them with the literature on the teacher's role in mathematical discussion.

The first significant outcome of this study is it reconfirms the essential role of emotion in the dynamic processes of discussion during kindergarten mathematics lessons. Ms. Kelly asserted that creating a respectful atmosphere should be the first and foremost step for promoting children's willingness to participate in mathematical discussion (O'Connell & O'Connor, 2007), and she emphasized her role in scaffolding students' emotional responses (Rosiek & Beghetto, 2009) during mathematics lessons. As “emotions are intrinsically linked with motivation” (Hannula, 2006, p. 224), it is common knowledge that students’ motivation to learn can proceed easily when they are ensured a respectful and supportable environment to express their thoughts, comfortably, to others (Chapin et al., 2003). There is numerous literature that emphasizes the importance of the role of a comfortable and respectful classroom culture and suggest guidelines to reduce students’ anxiety and increase their participation in discussion (e.g., Ball & Friel, 1991; Chapin et al., 2003; Cobb et al., 1998; O’Connell & O’Conner, 2007; Vacc, 1993). However, even these studies fall short of fully showing how a teacher might really implement their approaches to create a learning atmosphere of participation in early childhood education context. Yet, this case of Ms. Kelly offers a detailed description of how she implemented her own beliefs about the role of emotion into her mathematics classroom, and how she tried to support her young students’ emotion to encourage their participation in mathematical discussion when they gave wrong answers or disagreed with others.

Another main finding was the identification of the teacher’s role in laying down and maintaining ground rules for becoming better speakers and better listeners within a respectful atmosphere. As Chapin and colleagues (2003) indicated, setting the ground rules is a big part of preparation for supportive and courteous talk in mathematics. Particularly, Ms. Kelly pointed out that it would be important for many of young children not yet familiar with the ways of participating in the structures of classroom discussion. Studies of classroom discussion reveal that if primary school students have opportunities to talk without specific guidelines on how to talk, they might not know how to explain their mathematical ideas (e.g., Anthony & Walshaw, 2002). Some of them might “feel intimidated in peer-led discussions because dominant students remained in power” (Lewis, 1997, cited in Mcintyre, Kyle, & Moore, 2006, p. 43). For this, Ms. Kelly asserted her roles in introducing her expectations to her students about how to contribute to mathematical discussion and what to do as a speaker and a listener (Wood, 2002). She further argued that establishing ground rules for respectful talk seemed to be a long-term project across the academic year. Thus she tried to maintain the norms for discussion—established in the fall—throughout the year; she encouraged students to join in gradually rebuilding social norms for speaking and repeatedly reminded students of those norms. This finding highlighted the significance of the teacher’s efforts to maintaining ground rules for mathematical discussion across a year, and this maintenance aspect of the teacher’s role has gone unaddressed in the literature.

Furthermore, this study showed Ms. Kelly’s specific examples of how to enable students to become active and courteous participants in mathematical discussion. She did this by allocating individual students specific roles and assigning their relational rights and responsibilities. These were assigned relative to each student’s personality and learning level in a small group activity. A similar perspective, one in which students should come to take on the particular roles and social identities relevant to the moment, was illustrated in many studies based on Goffman’s (1981) notion of the participation framework. For example, O’Connor and Michaels (1996) argue that the teacher must “provide all children with access [the] roles [such as a competent hypothesizer, evidence provider, maker of distinctions, and checker of facts] in the context of school learning”

(O’Connor & Michaels, 1996, p. 64). And Goodwin (1990) claims that the teacher must provide the relational rights and responsibilities that go with participants’ own roles in a particular moment for social interactions (Goodwin, 1990). These arguments seem to support Ms. Kelly’s conception of participants’ roles in mathematical discussion. She explained the dual rights and responsibilities of speakers and listeners in the social context of classroom discussion. Students have the right to say their ideas comfortably and the responsibility to listen to others’ talks attentively. They also have the right to listen to others’ ideas so as to reexamine and expand their own ideas, and the responsibility of verbalizing their ideas to one another so as to clarify what they really understand and how they might rebuild their problem-solving strategies. In this vein, she particularly emphasized the need to provide young children more concrete guidelines of how, in the process of mathematical discussion, they should think and act as both speakers and listeners. She further asserted the importance of her role in observing attentively how young children’s discussion went and in scaffolding students’ talks appropriately. Thus, this study could be an illustrative example of the teacher’s roles in laying down ground rules for prompting young children’s engagements in respectful talk in the kindergarten context.

Lastly, this study revealed the kindergarten teacher’s instructional strategies that facilitated all the students’ equitable participation in mathematical discussion. According to Chapin and colleagues (2003), such participation has two aspects: “how to make it possible for all students to participate actively in the talk form time to time, and how to make certain that all students are listening actively all of the time” (p. 107). However, Ms. Kelly recognized such limitations as not being able to confirm whether every student spoke at least once every few mathematics lessons and who might minute by minute disengage or tune out from classroom discussions. In this respect, she developed her own strategies for minimizing such limitations, thereby maximizing the equitable participation for all students in her mathematics classroom. Her strategies, such as using easier problems for struggling students, assigning turns by calling on students, and utilizing partner talk, seem to be general approaches that have been used in many classrooms. Yet it is meaningful that those ways are feasible for early childhood teachers in their own classrooms. Furthermore, Ms. Kelly highlighted that the meaning of equitable participation did not mean the same amount of opportunities for individual students to talk in mathematical discussion. According to her argument, a teacher should basically provide chances, at least once a lesson, for all students to speak. Still, the teacher must also carefully think about how to offer one or more chances to struggling students reluctant to voice their ideas. From her viewpoint, the teacher should thus maintain a balance between providing all students with equal opportunities of participating in saying their ideas and creating fair participation for an individual student's contribution in classroom discussions. This view on the teacher's role for equitable participation was one that I had not come across in the literature about mathematical discussion (e.g., Chapin et al, 2003; Stein et al., 2008; Walshaw & Anthony, 2008).

To sum up this chapter on getting started with mathematical discussion, the findings from the qualitative case study revealed that Ms. Kelly, as an experienced kindergarten teacher, conceptualized her first role as creating a respectful learning environment for emotional scaffolding. In her view, students, when learning any subject matter, inevitably encountered various kinds of emotion (e.g., excitement, enjoyment, anxiety, and boredom). It was in such moments that Ms. Kelly used instructional strategies to motivate appropriately students' emotion to stimulate their deep involvement and persistence in mathematical discussion. In doing so, she asserted that her efforts for initiating mathematical discussion could gradually help not only encourage young children's willingness to participate in mathematical discussion, but also develop their ownership of their mathematical learning. However, even though this role of the teacher is necessary to enable young children to more motivate in mathematical discussion, it is not sufficient for students to become active meaning makers to construct their own knowledge in the path of their mathematics learning. As Anthony and Walshaw (2008) point out, "[a] pedagogical approach that is able to move students' thinking forward involves significantly more than developing a respectful, trusting, and nonthreatening climate for discussion and problem solving" (p. 527). Therefore, in the next chapter, I illustrate the findings of how Ms. Kelly conceptualized her role of orchestrating mathematical discussion in social and cognitive ways of scaffolding, and I discuss her instructional strategies to help develop young children's mathematical thinking and reasoning.

Chapter VI. Scaffolding Student Discussions

After creating a respectful learning environment for mathematical discussion, Ms. Kelly used a broad set of instructional strategies to scaffold student discussions. Chapter Six illustrates three major roles: (1) *purposefully planning discussion-intensive mathematics lessons*, (2) *scaffolding student talk to improve their mathematical thinking*, and (3) *maintaining the balance between flexibility and inflexibility*. After detailing each role, I will discuss the findings focusing on my research questions about a teacher's ways of thinking about her role as a facilitator in mathematical discussion, as well as her specific implementation of instructional strategies to socially and cognitively scaffold her young students' mathematical thinking and reasoning. Carried out through mathematical discussion, her scaffolding was intended to make the students active knowledge constructors in their mathematical learning.

PURPOSEFULLY PLANNING DISCUSSION-INTENSIVE MATHEMATICS LESSONS

Ms. Kelly believed classroom discussion was "very important in their math learning, because they have to be able to explain or verbalize things in life and to really understand it" (Interview transcript, 03/24/11) and so tried to find lots of opportunities for children to have discussions. However, she felt that increasing the amount of student-to-student and student-to-teacher talks in no way assured the development of students' mathematical thinking and reasoning. She argued that the teacher must first purposefully plan and carefully organize discussion-intensive mathematics lessons. This was the first step to engaging students in mathematical discussion and eventually enabling students to comprehensively understand mathematics content (Walshaw & Anthony, 2008). She emphasized that this type of preparation included coming up with *cognitively demanding tasks* and *talk formats* when planning mathematics lessons.

Cognitively demanding tasks

According to Ms. Kelly and as evidenced in her classroom, the effectiveness of classroom discussions depends highly on the quality of mathematical tasks. As Sohmer et al. (2009) point out, classroom discussions, not linked to academically rigorous tasks, could not produce learning gains. Ms. Kelly also recognized that mathematical tasks are central to students' learning and their participation in mathematical discussion. For example, at an early stage of a unit on fractions she began the session, by asking a following question:

Ms. Kelly:	Okay now, I have a little problem, and maybe you can help me with it. Now sometimes we will have a whole, but the whole is not going to be just one thing. It might be a group.
Students:	Oh…
Ms. Kelly:	And I have two packages of bean seeds and I have to divide these two packages. And I have to share it with all five classes, because that's all I had… Now if I had to share it with Ms. Morris and myself. That would be easy. I would give Ms. Morris one…
Students:	One package!
Ms. Kelly:	And I would keep the package it, seeds… But! We have to share two packages of bean seeds with five classes…
Students:	[Whispering] (Observation transcript, 03/22/11)

Ms. Kelly framed a whole-class discussion on this day with the fraction problem: how five kindergarten teachers could share two packages of bean seeds equally. She noted that this type of fraction problem, two divided by five, seemed to challenge young children to find problem solving strategies. Nevertheless, she argued it would be an appropriate topic for a whole-class discussion at the kindergarten level. This is because this task has important mathematics content and can be solved in multiple ways; it could be cognitive-demanding and challenging for young children (Hiebert & Wearne, 1993). Also, since it would not exceed the potential level of kindergartener mathematical development, she assumed that her students could, in the process of classroom discussion, handle this task (Moll, 1990). She further thought that a discussion could "allow students to approach the task in different ways before being guided by the teacher into explicit formulations or arguments" (Sohmer et al., 2009, p. 112).

Ms. Kelly also pointed out that the aforementioned problem pertained to children's daily school lives. It could thus help motivate students to engage more in a discussion as well as be stimulating (Amos, 2007; Varol & Farran, 2006). She addressed, "I just wanted them to see that we use fractions and we divide things out into equal groups in our everyday lives. So they are challenged because it is real and when they see math at work in their life, it is more impacting on their motivation to join in a discussion" (Interview, 03/24/11). Ms. Kelly's view aligned with how the literature defines meaningful tasks: mathematical tasks must be driven from students' real world contexts (NCTM, 1991; Nicol & Crespo, 2005) and be connected with their skills and knowledge they already possess (Hiebert et al., 1997). For this, she tried to play a role in appropriately exposing students to worthwhile mathematical tasks, connected with student's daily lives inside and outside of school and their prior experiences.

Talk Formats

Ms. Kelly also stressed planning appropriate formats for productive talks so that individual children could participate in discussions and explore key mathematics content. She used several talk formats, ranging from whole-class discussions to one-on-one discussions, depending on her instructional goals of each mathematics lesson. Each lesson usually consisted of two or three different types of talk formats. This was because each talk format had both strengths and limitations in guiding young children's mathematical learning (Chapin et al., 2003). Ms. Kelly thought that utilizing a variety of talk formats for one mathematics lesson could reach more children to express their own ideas and listen to others', as well as to be more interesting in mathematics activities. That is, when providing only whole-group discussions, an individual child would miss out on more chances to speak; when using only one-on-one discussions, a child would fewer opportunities to hear other classmates' different ideas. Moreover, she pointed out young children's short attention span and their tendency to lose focus. This would become a problem if each 35-minute mathematics lesson consisted of only one type of talk format (Interview, 03/24/11).

Whole-class discussion

The primary talk format was the whole-class discussion. Ms. Kelly asserted that the teacher should play a role in facilitating students' participating in whole-class discussions. This could be done through giving children opportunities to listen to others' ideas and to share different viewpoints of a mathematics problem.

> During our whole-class discussion time, the whole class comes together to, either to explain what ideas they have or how they did something. The reason I do that is because when children see their own peers understanding it, they're motivated to understand also and it's not just the teacher knows this. So that, they are learning from each other and also children can explain things in ways that other children understand better than sometimes adults can do it. So I use that that way. Also I call on some students that at times that try solving things in a different way, so we can see their different strategies, so we can have those different viewpoints and learn that we can solve things in more than one way. So I mean I use the students to be my teachers. (Interview, 03/24/11)

Ms. Kelly emphasized whole-class discussions as the social context for children to learn mathematics—through explaining their ideas and listening to their peers' ideas. She assumed that children within this type of talk format would be more motivated to understand mathematics than from listening to the teacher's explanation. In this respect, her role in whole-class discussion was not to deliver the mathematics content directly but to make a space for children to talk and share ideas (Chapin et al., 2003). Further, in the same interview (03/24/11), she pointed out that a whole-class talk format could be effective for discussing new mathematics concepts and to generate as many ideas as possible in the opening part of the lesson. Also, it could be appropriate to compare and contrast these ideas in the closing part of the lesson. This would narrow the focus for reaching the main mathematics content they should learn on that day (Sherin, 2000).

Teacher-guided small group discussion

On the other hand, Ms. Kelly asserted that small group discussions could be a proper talk format when they were followed by whole-class discussions. She also recognized that small group discussions could provide a space to meet the needs of low-performing students and high-performing students respectively (O'Connell & O'Connor, 2007). For this, she often divided students, based on their academic achievements in mathematics, into two small groups. For example, in the middle stage of a unit on number and operations, while half the class, the higher-achieving students, played mathematics games on the tables, she was working with lower-achieving students at the front of the room. She then switched. She gave five types of word problems:

> Comparison: Bela had 10 [20] hearts. David had 6 [9]. How many more hearts did Bela have?
> Addition: Hailey had 6 [9] pecan trees in the front yard and 6 [11] in the back yard. How many does she have altogether?
> Subtraction: Jacob had 12 [21] horses in the corral. 3 [9] jumped the fence and ran away. How many horses are left in the corral?
> Missing Start Addition: Kaylee had some balloons. Her sister gave her 3 [10] more. Now she has 9 [19] balloons. How many did she have to begin with?
> Missing Addend: Claire had 5 [12] cookies. Her mother gave her some. Now Claire has 12 [17] cookies. How many did her mother giver her?
> Missing Subtrahend: Noah had 10 [19] cookies. His little brother Harry ate some. Now Noah only has 3 [13] cookies left. How many did Harry eat?
> *Numbers in [ ] are for more skilled group students.
> (Teacher's math lesson plan, 02/23/11)

Ms. Kelly had student solve these problems using base ten counters, and then she had students talk about their ideas and share their solutions with the class. She pointed out that these types of mathematics problems were commonly observed in many kindergarten classrooms, but the procedure of the small group discussions with these problems could be highly influenced by the teacher's role in stimulating and supporting students' discussion. She illustrated her instructional purpose for it:

> I like to work with small groups. In this type group, I am in charge of the discussion, but it means I'm a leader. Instead, I want to guide students to engage in the discussion. So, I think it provides students with more opportunities to verbalize their thinking than other groups. And in small group activities, I use the same kind of math problem types usually, because students need to get experiences with all those different types of problems. But with my low performing students, I use the lower numbers, so they talk their ideas at their level. And with the higher performing students, I use the higher numbers, so they are still challenged and can be using their skills at their level. So I do it that way, so I can slow down, a little bit, and get thorough explanations for the lower performing students. And I can still challenge the high-performing students. (Interview, 03/24/11)

Ms. Kelly asserted that this sort of teacher-guided small group discussion allowed all students to have more opportunities to verbalize their thoughts. Within the same type of mathematics problem, she used the lower numbers for struggling students and the higher numbers for advanced students. She indicated that this talk format could help low-performing students engage their ideas at their own levels, as well as enable high-performing students to be challenged with chances to explore more difficult levels of tasks at their own levels (O'Connell & O'Connor, 2007).

Small group discussions focusing on student-to-student interactions

Ms. Kelly also emphasized the essential role of small group discussions in promoting communication between students. She pointed out that young children should have more opportunities to share their ideas with peers.

> Students learn between – from each other and we need to give students more time to verbalize their ideas, instead of the teacher just standing up there and verbalizing it for them. And so we had a group activity where we divided the class into four different small groups. I gave them a package of shapes and their job was to decide if the shape was easily divided into halves or could easily be cut or folded into half. And then they had to sort the shapes like that. And so the reason I do that kind of thing is for the kids to talk about things and solve it on their own. So, I also think it gives them chances to think more independently. (Interview, 03/24/11)

Ms. Kelly indicated that the teacher in this type of small group discussion could give students more time to talk about their ideas aloud, instead of providing answers or solutions directly. In this process, she asserted students could become more independent thinkers as they explored mathematical tasks. She further argued that the teacher's role should be essential to facilitating this type of small group discussion. Despite its benefits, she mentioned, "Small group discussions in kindergarten can be very difficult" (Interview transcript, 03/24/11). Her biggest challenge in conducting this talk format was how to manage her young children who might not be familiar with social norms for discussion. In this respect, her role was to guide kindergarteners with more concrete directions and instructions of how to cooperatively participate in this type of small group discussions (e.g., social norms for discussion, participants' rights and obligations), rather than whole-class discussions, teacher-guided small group discussions, and partner talks (See Chapter IV). She pointed out that while multiple conversations occur, the teacher should roam among the small groups, carefully listening to students' conversations, attentively monitoring students' progress at solving the tasks, and then give appropriate feedback or comments to expand each group's discussion (Interview transcript, 03/24/11).

Partner talk

The last format in facilitating mathematical discussion in Ms. Kelly's kindergarten classroom was partner talk. In this talk format, the teacher plays a diminished role and the teacher must carefully support students voicing their thoughts with actual words as well as by monitoring the partner talk to be productive (Chapin et al., 2003). Ms. Kelly often used this format to maximize opportunities for students to give voice to their thoughts. She asserted that speaking with their partners could "give the children more opportunities to speak, instead of having everyone speak just one at a time in a large group" (Interview, 04/07/11). She also thought that this talk format would need only a short time, one to five minutes. It would give a chance to everyone within the limited time of a mathematics lesson (approximately 30 to 40 minutes).

Ms. Kelly:	Now what we're going to have you do is you're going to go around the room somewhere. And… you're going to get some objects that you are sure are safe that would make up a good addition story. And then… let's see what I have in my plans, I'm going to have you, and find a partner and sit face-to-face, so this time it's going to be knee-knee-knee-face-face. Okay? Knee-knee-knee-face-face. Okay?
Students:	Knee-knee-knee-face-face!
Ms. Kelly:	And you're going to share your addition story with them. Okay? Now Please find the partner.

(Class transcript, 03/29/11)

Ms. Kelly initiated partner talk by saying, "find the partner," as described above, or "talk about it with the person next to you." She addressed:

> About the grouping of the students in this lesson, I wasn't considering students' achievement levels or personality of each student. I wasn't too worried about who was with who because it was such a base-level lesson.

> It was at a very intro-lesson that I thought everyone could be successful, so I didn't have to match with high-performing and low-performing students in that (Interview transcript, 04/07/11).

Ms. Kelly indicated that how to decide the pairing of students would depend on the level of the task. In this activity, she thus had students find their partners freely without any other directions, because the task for this partner talk would not be difficult for anyone. On the other hand, if the task level was higher, she often paired a low-preforming student with a high-performing student. She believed that matching heterogeneously according to academic abilities would be beneficial to both low- and high-performing students. That is, a low-preforming student was more encouraged to be involved in mathematical discussion with the partner's assistance; it helped a high-performing student to develop higher self-efficacy in cooperative learning situations (Stahl, 1994).

Furthermore, Ms. Kelly pointed out that partner talk was effective for passive students and for English language learners to actually engage in discussions. They could feel more comfortable in this manner voicing their ideas (Interview transcript, 04/07/11). To illustrate, this year she had one particular child who was learning English as a second language:

> I have one child in here—you probably saw one little girl, Bela, in the first group who had difficulty. She is an English language learner and she just came to this country the week before school started. She had had no formal school experiences or preschool experiences before (Interview transcript, 02/24/11)

Bela had arrived in the U.S. with her family the week before school started. She had never been in any formal schools before entering Ms. Kelly's class, and thus everything in a school was new to her. At the beginning of the school year, since she also had no experience learning English, she could not understand what the teacher and students said, she had not formed any bonds with the teacher and other students. Ms. Kelly, therefore, needed to do something special for her to be involved in the class as well as to learn English.

> And, of course, [Bela] had speaking issues, so at the beginning of the year, for her, I wanted her involved. I wanted her to learn how to speak, but you can't expect her to speak because she's still learning the language. She is in the silent period of that. And so I didn't have her speak much, but usually give her the same math lesson, we worked still on counting a lot. And we did have special activities for her, like language masters, so she could review things, because I don't want to have her excluded from the group. (Interview transcript, 02/24/11)

Ms. Kelly was aware that teaching and learning mathematics would not just be a problem of numbers; it included the language. She thus used different approaches for Bela depending on the stages of her language development. During the fall semester, instead of compelling her to speak much, Ms. Kelly exposed Bela to vocabularies and expressions that are in everyday use. While she had Bela participate in the same mathematics lessons with her peers, Ms. Kelly prepared special activities for her to learn the English language.

> Lately, she has been starting to speak more and she's counting more and she is more confident. So I write it in my math plans for her. As I am planning a one-on-one discussion, there is something in there that she can join in and learn from. (Interview transcript, 02/24/11)

From the beginning of the spring semester, although Bela's English level was still low, she learned some simple words in English and spoke in sentences. She also started counting more and became confident more in mathematics. Ms. Kelly thus utilized partner talk for Bela to engage in talking and to share her thinking, even if she used only a few words and simple sentences. She believed that, Bela, who was hesitant to speak out in front of the whole class, could have a chance "to practice [her] contribution with just one conversational partner" (Chapin et al., 2003, p. 20).

SCAFFOLDING CHILDREN'S TALK TO PROMOTE THEIR MATHEMATICAL THINKING

With carefully planning cognitively demanding tasks and appropriate talk formats, Ms. Kelly made conscious efforts to elicit, support, and extend students' talks in the interactive process of classroom discussions. She defined her role here as a facilitator who could scaffold children's talk through appropriate questions and responses.

> If I wasn't here as a facilitator, I think everything would be pretty random. And they do learn without me many times and I try to give them opportunities where they work with each other in free time and in free play, they are making these huge, elaborate, intricate structures with pattern blocks and they are using, like, radial symmetry and stuff like that and they're discussing things on their own. They still might be learning things about math, but they don't know where they are heading towards all the time. So I think they do need guidance, especially at this young age. And I think the teacher must facilitate their doing, thinking, and talking to meet the TEKS. Also I think the teacher needs to question and respond appropriately. So if we didn't have a teacher to scaffold them or assist them, we wouldn't be learning all those things that they need to be mastering at this age. So I'm there to promote their thinking, to get them to think about new things, to reinforce what they already think, to help them learn how to discuss things with a large group, with a small group, and even individually, too. (Interview, 04/29/11)

Ms. Kelly described the pivotal role, in student learning, of the teacher as a facilitator. She recognized that free time or free play without the teacher's direct intervention were necessary opportunities for students to learn mathematics by working and talking with each other. However, she asserted that pedagogical scaffolding through the teacher could allow students to learn important mathematics content in the kindergarten learning standards such as TEKS[6]. The teacher' role should be to guide and assist students through appropriate questioning and responding. Students could thereby improve their thinking, learn new mathematics contents, reinforce their prior knowledge, and develop their understanding of how to talk about their ideas. In this section, Ms. Kelly's roles of how to scaffold student talk are illustrated, focusing the four following aspects: (1) probing students' answers, (2) revoicing students' talks, (3) using wait time, and (4) stepping in and stepping out. Here I describe these major strategies that characterize the practices of Ms. Kelly I observed and interviewed, related them to the research literature, and illustrated them with examples from classrooms and teachers' reflections in interviews.

Probing children's answers

Ms. Kelly's questioning for probing students' answers was frequently observed in classroom discussions during mathematics lessons. The following episode shows how she could use probing questions when seven low-performing students discussed and shared their solutions of a mathematics problem with the others in a small group discussion.

Ms. Kelly: Okay let's try another one. This time Grace has two candy bars. Now her dad gave some more. Now Grace has five candy bars. How many did the father give her?

Students: ……

[6] TEKS is the state curricular standards, the Texas Essential Knowledge and Skills.

Ms. Kelly: Let's start with two. And her dad gave some more. Now she has five. So, how many did he give her?
Sarah: I think three.
Ms. Kelly: Three? Why do you think the answer is three?
Sarah: Um… First she got two [pointing to two Unifix® cubes in the upper line of her workspace]. Then…
Ms. Kelly: And then?
Sarah: Then [pointing each cube in the bottom line] one two three four five. So, [covering the first twos in both the upper line and the bottom line with her hand] her dad gave three [pointing the left threes in the bottom line].
Ms. Kelly: Oh, so you know that the dad gave three. That's good idea. Now, another way you might know too.
Matthew: Well… I'm not sure…
Ms. Kelly: But you knew that in the end. OK. Let's look at this [pointing to Matthew's solution]. Here is another way you can do too. Matthew, you're a good thinker. Could you please how you got that answer? Show how you did that.
Matthew: Um… I first count two [pointing the first two cubes], and then… I count these [pointing three cubes]…until five.
Ms. Kelly: Aha, you want to say you first count these two cubes, and then you counted the rest of three cubes until five. It is right what you are saying?
Matthew: Yes.
Ms. Kelly: Good job. So you think Grace had five in the end, right? And we knew that she had begun with two, right? That tell you how many her dad had given her.

(Observation transcript, 03/03/11)

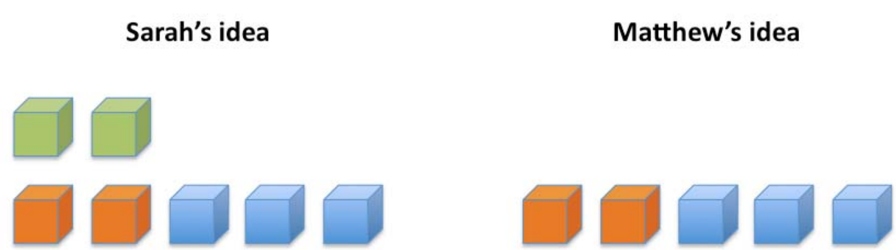

Figure 2. Two different problem-solving strategies

For some, this discussion between teacher and students may seem slightly laborious. However, Ms. Kelly recognized that for Sarah and Matthew to verbalize how to solve the problem and clarify their ideas would be a teachable moment. The rest of the students could hear two ways to solve the problem (See Figure 2). Ms. Kelly wanted Sarah, one of the low-performing students, to explain her idea to the others. While encouraging Sarah to verbalize her idea clearly, Ms. Kelly asked probing questions "Why do you think the answer is three?" This type of question helped Sarah's idea become more visible, because of "[prompting] a recall of facts" or "[stimulating] deeper thought about math ideas" (O'Connell & O'Connor, 2007, p. 19). Sarah used a total of seven Unifix® cubes to represent her idea. She put two cubes Grace first had in the upper line, and five cubes that Grace had after her dad gave some more in the bottom line. She then covered the first two cubes in the both top and bottom lines, figuring out the answer was three.

Ms. Kelly was aware that Matthew, on the other hand, had a different strategy. Sarah's strategy seemed to compare two numbers; Matthew's idea looked like subtraction-problem-solving strategy. He first put five-cubes, counted the first two cubes, and then counted the rest to five. The problem, however, was that he was reluctant to speak his idea aloud. Ms. Kelly encouraged him to talk about his idea, rephrasing his vague explanation, so as to enable the rest of students figure out what Matthew was thinking. Further, when Ms. Kelly asked Matthew how he got that answer, she was probing for his knowledge about this problem. She highlighted such questions as "Why do you think so?" and "How did you get the answer?" She stated,

> When I interact, I often ask why and how questions. It's very beneficial because… If they can explain how they solved the problem or why they think that, then they know it. If they can't verbalize it, then they probably don't understand it. So, to make sure that they really know it, I want them to be able to explain it. (Interview, 03/24/11)

Ms. Kelly's appropriate use of probing questions to understand student thinking could promote students to understand essential facts, and it could further provide students with opportunities to rethink deeply mathematics ideas (O'Connell & O'Connor, 2007).

Revoicing children's talks

Ms. Kelly frequently used the strategy of "revoicing," which was her repeating, rephrasing, and expanding students' talk (O'Connor & Michaels, 1996, p. 71). For instance, in the middle of a unit on numbers and algebraic reasoning, Ms. Kelly had the students demonstrate on dominoes their adding. With a data projector in the whole-class discussion, she showed a domino—three spots on one half and one on the other. She then asked students to think about how to count the total number of this domino faster.

Ms. Kelly:	Let's see. I can count one two three four [pointing to each dot]. But this way is pretty slow. What else can we do?
Ashley:	[Raising her hand.]
Ms. Kelly:	Ashley, do you have an idea?
Ashley:	[Nodding]
Ms. Kelly:	Come here and show how you could count faster.
Ashley:	I want to count like this. Three [pointing to a group of three dots] and four [counting one more dot].
Ms. Kelly:	Oh, you are saying that you want to count these three dots first and then count one dot, right, Ashley?
Ashley:	Right!
Ms. Kelly:	So, why do you think this way is faster than the way I counted?
Ashley:	Because I didn't count three, because I just knew that is three. I mean…

Ms. Kelly: OK, let me see if I understand… So you said that, like, you didn't even have to count that, like one, two, three [pointing each dot]. You just knew that instantly, didn't you?
Ashley: Yeah! I just was looking at it and I did it in my brain, so I instantly knew it was three.
Ms. Kelly: So, why did you count these three dots first?
Ashley: Because I think… I think high number is first counting, so I count three [pointing three dots of upper side] and four [pointing one dot of under side]. That's why I got the answer is four faster than you.

(Class transcript, 03/09/11)

After hearing Ashley's vague explanation about how to add the two sides, Ms. Kelly repeated it for the rest of the class to easily grasp what Ashley claimed, by saying, "So you said that, like..." as described above, At the same time, she asked Ashley to verbalize why her way would be faster and why she had counted three dots first. This gave Ashley the chance to clarify her own thinking as well as extend her mathematics reasoning. Moreover, the rest of the class learned the more effective strategy of counting on from a larger number rather than from first number.

Ms. Kelly pointed out that students, especially kindergarteners, would have difficulties putting their ideas into actual words clearly:

> After the children often say something, I will often rephrase it in a more understandable way… because sometimes when young children explain things they are not always linear in their thinking. So I want to rephrase it, so that they can hear it in a different way and it's more logical sometimes for them to understand it, and it helps for them to foster more explanations. And often the children are learning from each other. So I want the other children to hear it too. So I want to rephrase it, or sometimes I just want to repeat it, so that everyone else can understand it. (Interview, 03/24/11)

Ms. Kelly often used the strategy of rephrasing as a way for both the speaker and the rest of the class to understand the speaker's ideas more logically. She indicated that what young children explained was not always in accord with what they thought. She thus listened attentively to student talk and rephrased their explanations, promoting mutual mathematical understanding (Forman & Ansell, 2001). The strategy of revoicing can clarify for the speaker his/her ideas; it can help the rest of the class develop their understanding of the speaker's explanations. It also enables students to understand the necessary content of mathematics, to extend new ideas, and to further classroom discussions in mathematically enriching ways (O'Connor & Michaels, 1996).

Using wait time

As another strategy of scaffolding student talk, Ms. Kelly recognized the role of wait time when she interacted with the class as a whole. According to Tobin and Capie (1983), the meaning of wait time refers to the duration of silence between speakers during verbal interactions. Ms. Kelly tried to wait at least ten seconds, and she argued that wait time should be longer after posing high cognitive level questions to young children, as the following dialogue shows,

Ms. Kelly:	Okay, listen carefully and use your counters. And I'm going to be looking to see who thought of a good way to solve it and you'll get to share it after that. Okay, here we go. Jennifer had three hair bands. Her mother gave her some more. Now she has six hair bands altogether.
Students:	[Students starts to talk]
Ms. Kelly:	Don't tell me anything; show me how you're going to solve it. How many did her mother give her?
Students:	I did! I did! Three!

Ms. Kelly: Oh, take you time… Jennifer had three hair bands and her mother gave her some new ones, now she has six hair bands. You're going to show me with the counters.

Students: [Sounds of students' counting]

Ms. Kelly: [After waiting for approximately 20 seconds] Does anyone have a good idea?

Ethan: [Whispering to the teacher] I did! I did!

Ms. Kelly: [Quickly checking Ethan's work and smiling] Ooh, I saw a good way. Okay. Ethan. Everybody look at Ethan's area. Show us how you did it.

Ethan: I got three out and then I got another three out and then I put them together and…

Ms. Kelly: Well, that's not what really you did. Why did you get another three out? You had three out…

Ethan: Because I knew that three plus three was six.

Ms. Kelly: Ah! So he had three out and he said that she was going to have six hair bands after her mother gave her some. So he knew that he had to get three more out. So how many did her mother give her?

Students: Three.

Ms. Kelly: Okay, I'm going to look for a different way of doing it. Alexander, how did you solve it? Show me how you're going to solve it.

Alexander: Um… [Sounds of Alexander's counting]

Ms. Kelly: [After waiting for approximately 10 seconds] Okay, Andrew. How did you solve it?

Alexander: Um…

Ms. Kelly: [After waiting for approximately 20 seconds] I saw you had a good way of thinking. Move your container and show me, please. Alexander.

Alexander: Um… I got six out… And…

Ms. Kelly: You got six out… So?

Alexander: Um…

Ms. Kelly: [After waiting for approximately 10 seconds] Why did you six out?

Alexander: Because…

Ms. Kelly: [After waiting for approximately 10 seconds] Because… you wanted it to equal what?

Alexander: Six.

Ms. Kelly: And then?

Alexander: And I covered some up.

Ms. Kelly: Under your hand are the ones that were what?

Alexander: Three.

Ms. Kelly: Why did you cover up three of them?

Alexander: Um… because Jennifer had three to begin with. And…
Ms. Kelly: And?
Alexander: I got three.
Ms. Kelly: Those three is what?
Alexander: The ones that her mother gave her.
Ms. Kelly: Oh, that's a little bit different from how Ethan did it, too.
(Observation transcript, 03/03/11)

When Ms. Kelly posed a question, some students immediately raised their hands or started to give an answer. This scene is commonly observed in many kindergarten classrooms. She used wait time to allow students to think through the problem before hearing the answers of those who raised their hands immediately (Chapin et al., 2003). She also asserted that using wait time could give students, whether they had ideas or not, extra time to think of a solution before speaking their ideas aloud (Interview, 03/24/11). It was particularly effective for passive or silent students like Alexander. Ms. Kelly waited until Alexander was ready to explain how to solve it. Through her patient use of wait time, Ms. Kelly had Alexander make an important contribution, enabling other students to understand two strategies for solving this missing addend problem. As Tobin (1986) pointed out, such use of the pause in discussions can allow students "to consider what has been said and to assimilate new knowledge with previously learned information" (p. 192). Further, when the students are provided more wait time, the quality of the children's answers can improve (Labinowicz, 1985).

Stepping in and stepping out

As the discussion actively went on, Ms. Kelly was busy asking questions and responding to students' talk more sensitively. On the one hand, she needed to listen to students' ideas and to ask appropriate questions, so as to promote their participation in classroom discussions. On the other hand, she should coordinate students' different ideas and allow all students to reach focal points of the discussions. As such her dual role is aligned with Rittenhouse's (1998) description of the teacher's role of stepping in and out of mathematical conversations. Ms. Kelly used dominoes on a data projector and tried to get students to come up with ways to figure out what the missing addend (covered with a small sticky note) was (see Figure 3).

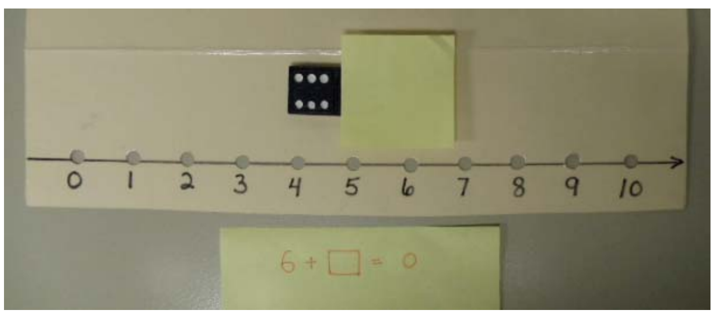

Figure 3. An example of the missing addend problems using dominoes

Ms. Kelly: Okay let's try another one. How many dots do we have on this side?

Students: Six.

Ms. Kelly: Six and we know we are going to have some over here, right? So we are going to put what sign over here?

Students: Plus.

Ms. Kelly: Plus, and do we know how many are under there?

Students: No.

Ms. Kelly: But we want to know, right? Hands down right there. So what I'm going to do is put a square there with nothing in it because I want to know what's in there. I'm going to tell you something about this domino. I'm going to tell you how many dots there are altogether on this domino and that will help us figure out how many are under there. There are ten dots altogether when you have this one and this one. So what do you think is going to go into that square?

Students: Four.

Ms. Kelly: Oh, excuse me. Take your time to think.

Students: …

Ms. Kelly: Okay, raise your hand if you think you know. Um… Anthony, what do you think goes in that square?

Anthony: Four.

Ms. Kelly: Now, would you come up and explain to us how you could have gotten four?

Anthony: Because…

Ms. Kelly: Go ahead. We'll wait…

Anthony: Um… Because six plus four equals ten.

Ms. Kelly: Well, how did you figure out that that was four? Show them.

Anthony: Okay. Because I know six plus four equals ten.

Ms. Kelly: How did you know that?

Anthony: I just knew it.

Ms. Kelly: You just knew that six plus four equals ten?

Anthony: Right.

Ms. Kelly: Ah, so you just had that number back in your head. Okay, that's one way. Um… If you know your number facts, you can figure it. But… what if you didn't know your number facts? What if you didn't know?... Okay, Noah, how would you solve?

Noah: I would solve it by, I would count and I would go, six…

Ms. Kelly: Okay, so you are going to start. Come up here and show us.

Noah: I count one, two, three, three, four, five, six, and then…

Ms. Kelly: Aha, and you think you're going to use your fingers too, maybe?

Noah: Yeah.

Ms. Kelly: Okay, show us how to… So you say six… We want to read what's in your brain. And you're going to go…

Jonah: [Folding each finger] And seven, eight, nine, ten.

Ms. Kelly: Ah, you take one finger at a time. Seven…

Students: Eight, nine, ten.

Ms. Kelly: So we know that there is ten, so what you do is start out?

Students: Six.

Ms. Kelly: Right, six. And then you count on – seven, eight, nine, ten. I love how Noah is doing it with us. Let's try it again. Six, seven, eight, nine, ten. So, what should go in this box here?

Students: Four.

Ms. Kelly: Four. Six plus four equals ten. Well are you ready to see if you're right or not? Okay. Why don't you uncover it? Were you correct?

Students: Yeah! So if you don't know those number facts, that can help you. (Class transcript, 04/14/11)

As Anthony started to suggest a solution, Ms. Kelly shifted back and forth between the role of participant and of commentator (Rittenhouse, 1998). While she carefully listened to Anthony's explanation, she attentively responded to him. At the same time, she tried to pose questions for Anthony to clarify his thinking by asking, "how did you figure out that?", as well as to remind the rest of the class to figure out what he was thinking. Furthermore, while Ms. Kelly encouraged Noah to say aloud his different strategy to other students by listening and responding appropriately, she reminded students of the focal points of this mathematics problem. This is because she believed, "just knowing the answer is not enough, so but knowing how you got the answer, I think, is more important" (Interview, 04/29/11). In this respect, Ms. Kelly was a participant, making students more conversational. All the while, as a commentator she would step out of the discussion, so as to articulate students' ideas, to allow all students to comprehend what was happening in the discussion, and finally to help them to develop their own mathematical thinking and reasoning.

MAINTAINING THE BALANCE BETWEEN FLEXIBILITY AND INFLEXIBILITY

Although Ms. Kelly tried to prepare well-planned lessons for mathematical discussion by selecting cognitively demanding tasks and appropriate talk formats, she knew her lessons did not always go as she planned. She often got unexpected replies from students. When student responses diverged enough from her lesson plan, she had to make decisions about where to go next in the discussion. She had to be flexible in handling the flow between expected and unexpected students' responses.

This section describes teacher's flexibility and inflexibility in the procedure of mathematical discussion in terms of three aspects: (1) a planned mathematics lesson, (2) moment-by-moment decision making in the midst of the lesson, and (2) narrowing the scope of the students' thinking. Consider the episode described above of finding problem-solving strategies for the fraction task of equally sharing two packages of bean seeds with five kindergarten classes. The whole-class discussion here is analyzed in greater detail to describe how Ms. Kelly flexibly or inflexibly made decisions between expected and unexpected students' replies.

The use of flexibility and inflexibility come from Leikin and Dinur (2007, p. 330)'s description of teacher actions in classrooms. Teacher flexibility is defined as the teacher's action "at a particular point of the discussion if he/she adjusts the planned learning trajectory according to student replies that differ from those he/she had foreseen." They consider the teacher's action inflexible if "a teacher does not make such adjustments or restrains the student's independent thinking," in order to complete what was planned. These definitions from Leikin and Dinur (2007) are derived from Simon's (1995, 1997) Mathematical Teaching Cycle. Simon (1997) illustrates that an actual teaching cycle involves the tension between responding to students' expected and unexpected responses and managing lessons according to the teacher's plan.

At an early stage of a unit on fractions, Ms. Kelly planned to have students discuss the problem of how five kindergarten classes could share equally two packages of bean seeds (see Figure 4). Ms. Kelly stated, “It was an introductive kind of lesson and it applied to what we were learning about fractions” (Interview, 03/24/11). She thought the problem related to students’ school lives and would thus motivate them to actively engage in mathematical discussion. She also expected them to easily understand a fraction defined as any number of equal parts, and thereby applying its definition to what they would be learning in the second part of the lesson that day (Interview, 03/24/11).

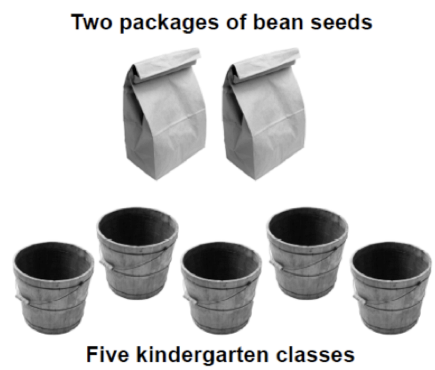

Figure 4. The fraction problem for the whole-class discussion

Moment-by-moment decision making in the midst of the lesson

Ms. Kelly first showed two packages of bean seeds to her students in the whole-class discussion. She told them that every kindergarten class needed some of the seeds and that every class needed to have the same amount of the seeds. She asked student to figure out how to share those seeds with five classes and then continued the discussion as follows.

Ms. Kelly: Who has an idea?

Stella:	Maybe two three…
Ms. Kelly:	Two three… Mm-hmm... Stella? Can you explain a little more about what you are thinking?
Stella:	I mean… one package for Ms. Morris and Ms. Anderson, and one package for Ms. White and Ms. Ball and you…
Ms. Kelly:	Aha, is what you are saying one package for two classes and one package for three classes. It is right, Stella?
Stella:	Yes.
Joshua:	But, I think that's not good.
Ms. Kelly:	Why do you think so, Joshua?
Joshua:	Because that's not a fair share.
Ms. Kelly:	Oh, not a fair share… Why do you think it's not a fair share?
Joshua:	I think… Because… If so, Mm-hmm… Two classes have one package and three classes have one package.
Ms. Kelly:	Aha! Let me see if I understand… So you said that, like, if so, Ms. Morris and Ms. Anderson have more seeds than the other three classes. So you think it is unfair?
Joshua:	Right! (Observation transcript, 03/22/11)

According to her lesson plan, Ms. Kelly's original intention of this lesson was to encourage students to verbalize their thinking and to freely share ideas. At this early point of the discussion, she felt things were going smoothly: Stella started to talk about her idea, one package for two classes and one package for three classes, and Joshua pointed out that Stella's idea would be not a fair share. Ms. Kelly reported that she could manage the whole-class discussion more effectively when the first speaker gave a wrong answer or an incomplete idea. She believed that this was because a wrong or incomplete idea motivated students to debate a focal point of the problem and expanded the conversation between students. Thus she asked whether Joshua's idea of a fair share would be supported by reasonable understanding or not. At the same time, she paraphrased and emphasized Joshua's explanation in order for students to focus the discussion on "fair" as key to solving this problem. It could also enable students to move the discussion in another direction with new ideas (O'Connor & Michaels, 1996).

Things, however, failed to proceed smoothly:

Ms. Kelly:	Oh, that's a good point. So, how can we share two packages of seeds equally?
Students:	…
Ms. Kelly:	[After waiting for approximately 10 seconds] Does anyone have a good idea?
Students:	…
Ms. Kelly:	[After waiting for approximately more than 10 seconds] It's hard for me too. Um… Can anyone help me solve this?
Students:	…
Ms. Kelly:	[After waiting for approximately 20 seconds] Hmm… Hmm… OK! So what I'm going to do is take the seeds and I'm going to pour them out here [in front of the visual presenter or doc cam] and I'm going to take the other package's seeds. [Students see what she is doing through the visual presenter.]
Alexander:	That's a lot.
Ms. Kelly:	Right, it's a lot. Now we have to divide them into five equal sets. Does anybody have any ideas about how I could do that? I'll get the bags. Who has an idea?
Students:	[Whispering to the teacher]
Alexander:	I have no idea.
Jonathan:	[Raising his hand] Oh, oh… Me, me, me!
Ms. Kelly:	Don't tell me anything. Let's think about it for some more seconds… [After waiting for approximately 20 seconds] Okay, does anyone have a good idea?

(Observation transcript, 03/22/11)

In spite of Joshua's idea about a fair share, Ms. Kelly saw that no hands were raised. She used wait time several times, and yet the students were still reluctant to suggest any ideas. She had not anticipated this situation. When she planned this lesson, she recognized that this level of mathematics problem—two divided by five—might be difficult for kindergarteners, even that students might not know how many seeds were in the two packages. The unforeseen student silences made Ms. Kelly use her flexibility and shift gears. She decided to pour the seeds out. And then, the students began talking again.

Ms. Kelly continued the discussion as follows:

Ms. Kelly:	Ooh, Jonathan? Do you have an idea?
Jonathan:	Give a seed to every person in the kindergarten.
Ms. Kelly:	You want to say to give a seed to every child?
Jonathan:	Yeah, like one by one by one…
Ms. Kelly:	You are a good thinker. But… Well, I don't know if we have enough time to do. But remember, I just have to have a fair amount of seeds for Ms. Morris, the same amount for Ms. White, the same amount for Ms. Ball, the same amount for…
Students:	Ms. Thompson!
Ms. Kelly:	Right! Mrs. Thompson, and the same amount for me. So how many groups is that?
Students:	Five.
Ms. Kelly:	Five! Five groups… What would you do, Daniel?
Daniel:	I would just take the whole bag and pass one down to every class at a time.
Ms. Kelly:	What? In one of these bags? But how do I know how much to put in each bag?
Daniel:	No, I meant like…
Ms. Kelly:	Oh! You had an idea.
Daniel:	Pass them one at a time to every class.
Ms. Kelly:	That's a good idea. Hmm… Hailey, do you have a different idea?
Hailey:	Yes, I would put five in each bag because…
Ms. Kelly:	Also, I have five bags here.
Hailey:	No. I meant five seeds in each bag.
Ms. Kelly:	Oh, and then you do five in each bag and then five in each bag again.
Hailey:	Yes.
Ms. Kelly:	Okay, I see. Hailey said that we put five seeds in each bag, and then five seeds in each bag again. That's another good idea! And I think both you [pointing Daniel and Hailey] have the similar idea. But, Daniel wants to put one seed in each bag, and Hailey wants to put five seeds in each bag. Is it right?
Students:	Right.
David:	[Raising his hand]
Ms. Kelly:	Oh, David, do you have a different idea?
David:	No…

Ms. Kelly:	Do you want to add something?
David:	Why don't you give them five each?
Ms. Kelly:	Why do you think so?
David:	Because I would say five seeds is faster.
Ms. Kelly:	Oh, so why do you think five seeds is faster?
David:	Well, because five is bigger than one.
Ms. Kelly:	Aha, is what you are saying that passing five is going to be faster than passing one. Is that right?
David:	Yes. (Observation transcript, 03/22/11)

As described above, Jonathan started to suggest the solution, but Ms. Kelly did not accept his idea. At first glance, Ms. Kelly's reply appeared to be flexible and supportive of Jonathan's thinking. She carefully listened to his explanation and rephrased it. However, instead of moving the discussion toward his answer, she tried to remind students that five groups should share these seeds equally. On the other hand, Ms. Kelly thought that Daniel's idea would not have worked in easily, but it would be a starting point to invigorate this discussion. She pointed out, "The idea came from Daniel eventually let some of those kids have those higher-level ideas" (Interview, 03/24/11). That is, based on Daniel's ideas, Hailey suggested putting five seeds in each bag. David further added his explanation of why Hailey's idea would be a better solution, rather than to put them in one by one. In this case, Ms. Kelly did not anticipate Hailey and David's ideas, but she decided to follow them. She figured it would be more beneficial than her planned paths.

> Discussion is used to get ideas across for me to teach them and for them to add their perspective on it. So I might give them an idea in one perspective or one way, but they might have other ways that are more effective for the other children to reach the mathematical goals of the lesson. (Interview, 05/19/11)

At this point, Ms. Kelly paraphrased and summarized these two students' ideas for the rest of the class to understand where the discussion was going.

After hearing David's explanation of why putting five seeds in each bag would be faster, Ms. Kelly moved on to invite all students to agree or disagree with his claim:

Ms. Kelly:	That would be fair, I think. Do you agree or disagree with that? Thumbs up if you agree. Thumbs down if you disagree.
Students:	I agree.
Ms. Kelly:	Can anyone tell us why you agree with what David said? Julia?
Julia:	Because I think that [pointing each bag] five, five, five, five, five is faster than one, one, one, one, one, one.
Ms. Kelly:	Aha, you agree with him because putting five in each bag is faster that putting one in each bag, don't you?
Julia:	Right.
Ms. Kelly:	Okay. You know what. We'll do that up here. So we have to have five. We have five teachers, right?
Hailey:	Yes, right.
Ms. Kelly:	So, Hailey said she would start with five, so five for… We'll call that our pile and then Ms. Morris' pile and then Ms. White's pile and then we put Ms. Ball's pile and then Ms. Thompson's pile. [Ms. Kelly put five in each bag, and students see these through visual presenter.] Let's make sure that each pile has five…
Students:	Right.
Ms. Kelly:	So we've got five, five, five, five, five, [as pointing each bag from left to right]… [as she points the rest of the seeds] But we have all of these left; we need to divide all of these…so what should I do?
Andrew:	What about twenty-two every?
Joshua:	That's too many.
David:	Just do five.
Anthony:	Five more again.
Ms. Kelly:	So I'll do five again?
Students:	Okay.
Ms. Kelly:	Okay, so five more. Let's do fives until we don't have enough fives. So we will do five more . . .
Julia:	Until it's done.
Ms. Kelly:	And five more and then five more and…
Students:	Five more

Ms. Kelly: Five more and five more. We still have plenty of seats, don't we? So we will do five more again. Five more, five more, five more, five more, and five more. Do you think we still have room for five more again?
[Some students say yes, some students say no.]
David: No!
Jessica: I think we do. I really do.
Ms. Kelly: Ok, let's put five in each bag. Five and five, five, five, five. Now, do we have enough for five more each, do you think?
Julia: We can think. How about two more?
Ms. Kelly: Oh, do you suggest two more? Do you agree or disagree with her?
Students: I agree!
Ms. Kelly: So we can do two. So let's do two. Two for them, two for them, two for them, two for them. Oh, should I keep going?
Students: Yes, two more then, two more, two more, two more. Now how many more for each?
David: Oh, one more.
Ms. Kelly: One more? One, one, one, one, one. And how many do we have left?
Students: One!
Ms. Kelly: One left over. Hmm… You know what? Matthew, it's like a magic bean seed. You can have it. Plant it outside your window and if we don't hear from you, we'll know you're at the castle, okay?
Matthew: Okay!

(Observation transcript, 03/22/11)

Ms. Kelly asked the whole class if they agreed with David or not. This strategy helps the teacher make sure students' understand what the discussion is about, before allowing the discussion to move to the next step. Also, by asking students about the reason for their agreement, Ms. Kelly can check whether their agreement is supported by a correct understanding. Such a strategy of asking for agreement is useful in eliciting student reasoning about the claim (Chapin et al., 2003), and it further helps other students reevaluate and rethink their own ideas (Chazen & Ball, 1995). She continued to ask students to verbalize their different ideas about how to divide all of these seeds until none were left. In doing so, her questions and responses to keep the discussion going enabled students to more actively engage in discussion. Here, Ms. Kelly directed the discussion back to her planned paths.

Narrowing the scope of the students' thinking

After putting all the seeds into five bags equally, Ms. Kelly was ending the first part of the mathematics lesson on this day, with a review of what students had discussed:

Ms. Kelly:	OK. Let's see… Did we divide something into equal parts?
Students:	Yes.
Ms. Kelly:	Yes, we did. So... Is that shared equally?
Students:	Yes.
Jacob:	It's a fair share.
Ms. Kelly:	Jacob, would you say it's a fair share?
Jacob:	Yes.
Ms. Kelly:	So, do you think Ms. Morris would be happy if I gave her this bag?
Students:	Yes.
Ms. Kelly:	How about Ms. White? How about Ms. Ball and Ms. Thompson?
Anthony:	Yes, they would be happy.
Ms. Kelly:	Really? Anthony, why do you think so?
Anthony:	Hmm… cause' they have the same number of seeds.
Ms. Kelly:	I agree. Anthony said they would be happy… because they have exactly the same amount of seeds. Do you agree with that? Thumbs up if you agree.
Students:	I agree!
Ms. Kelly:	Do you agree with that? Ashley, you don't agree?
Ashley:	Oh, I do agree.
Ms. Kelly:	Could you tell us why you agree with him?
Ashley:	Because... they have seeds equally.
Ms. Kelly:	Okay, that's a fair share!... Did we divide something into equal parts?
Students:	Yes.
Ms. Kelly:	Yes we did. It wasn't the same as one popsicle or one cob of corn, was it?
Students:	No.

> Ms. Kelly: It was dividing a big group of something into parts. . . . about a special equal part… Okay, next we are going to… Let's see… Hmm. I brought a pie pan. I like to imagine there is a big coconut cream pie in here. And Kaylee and I are going to share this pie equally. Now, Kaylee, should I draw a line here and you can have this piece and I can have this piece?
>
> (Observation transcript, 03/22/11)

Ms. Kelly asked students to review what they discussed and they learned, by asking, "is that shared equally?" Jacob reminded the rest of the class of the term of a fair share, and Anthony explained five teachers would be happy "cause' they have the same number of seeds." Ms. Kelly also moved on to invite all students to agree or disagree with Anthony's claim, so that the students would be able to reach the conclusion that a fair share meant everyone should have the equal amount of seeds. Further, she wanted that students could connect what they discussed with what they should learn about fractions on this day.

At this point, students had opportunities to rethink the meaning of fairness in terms of sharing the same amount of objects with others. According to Chapin et al. (2003), this is an important part of classroom discussion, in that "so much goes on during a class where student are using talk that is it important to leave students with focal points of the discussion" (p. 185). Through summarizing both small points and major conclusions after the discussion, Ms. Kelly helped students know that a fraction represented any number of equal parts, as well as applying its definition to what they would learn in the following part of the lesson about a half.

The second role of the teacher in mathematical discussion is scaffolding student discussions. According to Ms. Kelly, even though a respectful learning atmosphere is necessary to enable young children to more participate in mathematical discussion, it is not sufficient for students to become active meaning makers to construct their own knowledge in the path of their mathematics learning. For this, mathematical discussion in Ms. Kelly class relied on (1) *purposefully planning discussion-intensive mathematics lessons*, (2) *scaffolding student talk to improve their mathematical thinking*, and (3) *maintaining the balance between flexibility and inflexibility*.

Ms. Kelly's emphasis on appropriately select mathematical tasks in the planning of discussion-intensive mathematics lessons mirrors important work in recent pedagogical studies. As Henningsen and Stein (1997) pointed out, "the nature of tasks can potentially influence and structure the way students think and can serve to limit or to broaden their views of the subject matter with which they are engaged" (p. 525). However, Lee and Ginsburg (2009), through a lot of in-depth interviews and observations, reveal that many early childhood teachers in the United States "often limit their focus to one-to-one correspondence, simple counting and numbers, and perhaps naming and sorting simple shapes, even when children are capable of learning far more complex content" (p. 39). Accordingly, teachers may have a narrow understanding of mathematical tasks, and they may favor to simply provide mathematical problems of counting or sorting, listening student responses, and evaluating their correctness. This may happen despite many national, state, and local organizations emphasizing the importance of classroom discourse in students' mathematical leaning and development (e.g., NCTM, 2000, 2006; NAEYC, 2002). On the other hand, the current case study suggests the possibility that a kindergarten teacher can motivate young children to think, explore, reason, and discuss their ideas by posing appropriate mathematics tasks. Ms. Kelly intentionally chose cognitively demanding tasks that would covey the significant mathematics content and would be based on the students' interests. Furthermore, it would be relevant to five- and six-year-olds student's real lives and their prior knowledge (Amos, 2007; Hiebert et al., 1997). These worthwhile mathematical tasks could not only increase students' motivation to engage in mathematical discussion, but also encourage them to think about problem-solving strategies in new ways (Hiebert & Werne, 1993; Sohmer, et al., 2009).

The findings also show that the teacher in different talk formats performs particular roles in facilitating students' mathematical discussion. It supports the earlier work reported by many studies (e.g., Chapin et al., 2003; O'Connell & O'Connor, 2007; Sherin, 2002). On the other hand, detailed descriptions of Ms. Kelly's conception and practice about talk formats offer specific implications on how to select and use talk formats at the kindergarten level of mathematics lessons. She tired to purposefully organize lesson plans with two or three different talk formats, chosen according to mathematical tasks. Each talk format—whole-class discussion, small-group discussion, and partner talk—has its own strengths and limitations. Also, Ms. Kelly is aware that the length of young children's attention span is not long; so that each 35-minute mathematics lesson should consist of two to three talk formats. She had difficulty managing discussions if all the students were divided into small groups of four to six or if she were trying to focus the conversation between peers in a non-teacher-directed format. This is because kindergarteners are inexperienced at taking part in this type of small group discussion. They must still learn how to listen to others' opinions, wait their turns, and defend their ideas. Hence, the teacher needs to carefully plan and facilitate mathematical discussion by providing each participant with more detailed directions and expectations. This highlights her efforts to consider developmentally appropriate approaches (Copple & Bredekamp, 2009) so that the purposeful use of talk formats benefits young children's mathematics learning.

As another main finding, the data raises evidence across multiple sources that Ms. Kelly is an excellent teacher who is able to scaffold student talk in mathematics lessons. The quality of mathematics tasks and the appropriateness of talk formats are, of course, necessary to help promote students' participation in mathematical discussion (Sohmer et al., 2009). Nevertheless, simply providing them is insufficient to encourage students' exploratory talks (Mercer, 1998) and to move students' mathematical thinking forward (Walshaw & Anthony, 2008). To ensure this end, Ms. Kelly as a facilitator tried to guide and support students' talk, by questioning and responding to promote students' mathematical thinking in the procedure of classroom discussions. The findings illustrate her instructional strategies, such as probing students' answers, revoicing students' talk, using wait time, and stepping in and out of mathematical discussion. These kinds of pedagogical approaches are aligned with ones evidenced in the literature (e.g., O'Connor & Michaels, 1996; Rittenhouse, 1998; Tobin & Capie, 1983), although she never referred any research or noted similarities herself. My observations and interviews reveal that she skillfully and appropriately used those strategies with on-the-spot decision-making. Particularly, she effectively integrated various strategies for classroom discussions into everyday mathematics lessons, thereby familiarizing young children with talking, listening, and sharing their mathematical ideas. The detailed descriptions of this qualitative case study provides early childhood teachers with more resources and information about how to implement effective strategies for scaffolding young children's talk in their mathematics classrooms.

Last, this study underlines the need for early childhood teachers, in orchestrating mathematical discussion, to strike a balance between flexibility and inflexibility. Chapin et al., (2003) argue that most teachers often need to revise, modify, or improvise their lesson plans if students respond either more or less than was expected. For this, Ms. Kelly pointed out the teacher's duel roles in facilitating mathematical discussion. On the one hand, Ms. Kelly tried to accept all the students' answers and flexibly change her planned lessons when encountering unforeseen responses or situations. In such instances she departed from her planned paths, thinking it could be more effective for students to engage actively in discussions. On the other hand, Ms. Kelly argued that it is important for the teacher not to lose sight of mathematical goals of the lesson and to be inflexible in ultimately directing the discussion back to the planned paths. Many early childhood teachers are afraid to use classroom discussions in their mathematics lessons. They are uncertain about how far to follow students' ideas that ill suit their original plans. Further, they struggle to manage classroom discussions when young children's ideas are not related to mathematical issues or discussion topics. Ms. Kelly pointed out that the teacher should enable students to eventually reach the mathematical goals of the lesson, while facilitating flexibly and supportively unexpected student ideas. The result reported in this study supports the view of teacher flexibility that found in the literature. Simon (1997) argued, “Teaching is inherently a challenge to find appropriate balance between these two poles” (p. 76). Simon is referring to a balance between following students’ ideas that depart from the teacher’s notions where the classroom activity should go and managing discourse to focus on particular mathematical issues. Particularly, as Leikin and Dinur (2007) pointed out, “It is important that a teacher be able to judge when to be or not to be flexible, namely inflexibility is not always a negative characteristic of a teachers’ professionalism” (p. 330). Consequently, “The teacher may lead students to different outcomes flexibly with different scopes or guide them in the planned direction by narrowing the scope of the students’ thinking” (p. 343). The current study confirms the notion that mathematical discussion is not to increase the amount of talk between the teacher and students, but to motivate students to participate in the social process as well as to engage in the mathematics content that they should learn. The teacher should therefore make reasonable decisions that strike a balance between flexibility and inflexibility when dealing with unexpected student responses.

This chapter, from an analysis of interviews with Ms. Kelly, illustrated how she thought about her role for purposefully planned discussion-intensive mathematics lessons, orchestrating student talks to improve their mathematical thinking, and maintaining a balance between flexibility and inflexibility in the midst of the discussion.

The detailed descriptions, collected through classroom observations over a period of 14 weeks, also showed that how she tried to socially and cognitively scaffold student discussions by using a broad set of instructional strategies to develop young children's mathematical thinking and reasoning. However, she encountered challenges as she planned and facilitated the classroom discussion as the core way of teaching mathematics in her kindergarten classroom. In the next chapter I illustrate these challenges focusing on a teacher's thoughts and moment-by-moment decision-making on how to facilitate classroom discussions in every mathematics lesson.

Chapter VII. Overcoming Challenges to Mathematical Discussion

Although Ms. Kelly tried to create a discussion-intensive mathematics lesson that enabled young children to engage more in talking and sharing their ideas (see Chapter IV) and to use instructional strategies to scaffold young children's talk and develop their deep understanding in mathematics (see Chapter V), she also articulated and acted against what she saw as barriers to mathematical discussion with young children. Ms. Kelly explained, "I'm working on children becoming better listeners and better speakers." She conceded, however, that it was "not perfect in my math classroom by any means" (Interview, 04/07/11). She encountered challenges as she planned and facilitated the classroom discussion as the core way of teaching mathematics in her kindergarten classroom.

This chapter illustrates four different types of challenges that Ms. Kelly needed to meet in order to hold discussion-intensive mathematical lessons: (1) *the duality of a teacher's beliefs of discussion depending on mathematics content*, (2) *the limitation of a teacher's knowledge of content and students*, (3) *a tight daily kindergarten schedule within mandatory standards,* and (4) *parental expectations induced by pressures of high-stakes standardized testing*. The first two are what Ms. Kelly faced when using discussion across multiple types of mathematics content. The second two are what she experienced when using discussion despite direct instruction pressures of high-stakes testing in public school environments. Following a detailed description of each challenge, I then discuss the findings pertaining to the research questions of how the teacher recognized her own challenges to mathematical discussion and how she facilitated connections between her conceptions of mathematical discussion and the complexity of the current public school system.

DUALITY OF A TEACHER'S BELIEFS OF DISCUSSION DEPENDING ON MATHEMATICS CONTENT

The first challenge to mathematical discussion was how Ms. Kelly's own beliefs regarding an effective instructional strategy differed depending on the mathematics content. Despite Ms. Kelly's emphasis on the role of classroom discussion in children's learning, she did not always use discussions in teaching mathematics. On the one hand, she skillfully encouraged children to talk, listen, and share their ideas. She did this when the teaching of mathematical knowledge and skills were linked to algebra, geometry, measurement, data and probability. This way the children were encouraged to explore problem-solving strategies, to apply reasoning, and to develop mathematical arguments. On the other hand, when she was teaching knowledge and skills that were linked to basic number facts and basic operations, she tended to use direct instruction, rather than teacher-facilitated discussions. In this section, each classroom vignette is used to illustrate why it was difficult to use discussion in teaching all types of mathematical content.

In the last part of the unit on addition, after reading the book *Dominos Addition*, Ms. Kelly introduced how to write vertical addition equations using dominos.

Ms. Kelly: Yes. This is called domino addition. Learning to add is fun, especially when you use dominoes. It's easy. Let's learn how. Dominoes have to halves. Each half is zero, one, two, three, four, five, or…

Students: Six.

Ms. Kelly: Spots. The domino can have three spots on one half and zero spots on the other half, or two spots on one thing four spots on the other half, or even six spots on one half and…

Students: Six spots on the…

Ms. Kelly: …other half. Here is a complete set of dominoes. This is what it is supposed to look like. Add the number of spots on the top half of this domino to the number of spots on the bottom half. Now look at how they wrote these sentences. What looks different about them? Jennifer?

Julia: Because it's like the domino has…

Ms. Kelly: Look at the numerals. What's different?

Julia: Because it has zero then it happens zero plus zero plus zero…

Ms. Kelly: Okay, well... Instead of going across horizontally, where do they go? What is that word when they go up and down like this? A big word, it starts with a "V"?

Students: Vertical.

Ms. Kelly: Vertically, yes. We can write addition problems vertically and that is what we are going to be working on today – is writing addition problems vertically… So let's read this one and if we read it, then I will know you know how to write it. Zero and then plus is in the middle and it's right in front of that second number… Zero and that one line means equals zero and that is your sum, is down here… Look at how all the numbers are lined up right above each other in a straight line. Do you see that?

Students: Yes.

Ms. Kelly: When we write our problems later on that is what we are going to be doing. So let's read one more time. At the number of spots on the top half to the number of spots on the bottom half and, Matthew, let's read it.

Matthew: One plus zero equals one.

Ms. Kelly: Good. One plus zero equals one and look at it. See the domino? The one is on top; see there is the one, is the top numeral, zero is here, zero is the next numeral equals, I mean, we want to know what it is altogether and what is it?

Students: One.

Ms. Kelly: One. Come find the domino that has a total of one spot. Eagle eyes. Add the number of spots on the top half of each

> Domino to the number of spots on the bottom half of each domino. Let's read them together. One plus zero equals one. (Classroom transcript, 04/12/11)

Ms. Kelly said that dominos could be an appropriate material for introducing kindergarteners to vertical equations. She thought that the disposition of a domino, with spots on its top and bottom halves, bore a similarity to vertical equations (Interview transcript, 04/29/11). At first glance, it seemed as though she would provide a lecture on how to write vertical addition equations using real objects in a fun way.

The dialogue, however, began to show that Ms. Kelly's explanation was rather long and the children's answers were short – just yes or no. When Ms. Kelly showed the dominos, she might have been expecting the children to imagine number sentences, but the dominos did not remind the children of vertical equations at all. When she encountered unforeseen responses such as Julia's, Ms. Kelly ignored them, and directly asked for the terminology, hinting at a word that started with a "V." She then explained how to write addition problems vertically.

Ms. Kelly appears to have assumed that direct instruction might have worked well for introducing how to write vertical equations, whereas a discussion-intensive approach here might have confused young children first learning new mathematical skills (Interview transcript, 04/29/11). She directly instructed students how to write vertical equations, making the explanation short and clear. However, according to Ma's (2010) perspective of profound understanding of fundamental mathematics, direct instruction is necessary to clarify how to write equations, but it is not efficient to teach new mathematical skills. Ms. Kelly offered no explanation for why one should write vertical equations in that specific way: why the plus sign is in the middle of the equation and in front of the second number, why one line means an equal sign, why the sum is below that line, why all the numbers are in a straight line, and more generally why the students ought to learn vertical equations.

On the other hand, Ms. Kelly used the talk format of whole-group discussion for comparing vertical equations with horizontal ones. She called on two children to write two equations on the blackboard, one a horizontal equation and the other a vertical. She asked all the children to discuss the ways to improve the equations these two children wrote. She continued the lesson:

> Ms. Kelly: Now what we are going to do is I'm going to have someone come up here, someone who thinks they can already do this and they are going to write. I have two people, one person is going to write a problem horizontally – that means this way – and another person is going to write it vertically. Let's see, who wants to write a horizontal one?
> Students: [Some students raise their hands.]
> Ms. Kelly: Grace? Come over here and you're going to write a horizontal one. And who wants to write the vertical one? Joshua is next. Grace, get your chalk and yours are going to go across.
> Grace: Yes…
> Ms. Kelly: And Joshua, yours are just going to go vertically.
> Joshua: Okay!
> Ms. Kelly: Now remember about lining up those numerals, all right? Okay, let's… Now add the numbers of spots. [After choosing one domino] Okay, listen to the problem… four plus zero, show us that… Now show us four plus zero.
> Grace & Joshua: [After listening to the problem, they write the addition equations on the board respectively. See figure 5.]

$4 + 0 = 4$	$\begin{array}{c} 4 \\ + \\ 0 \\ \hline 4 \end{array}$

Figure 5. Grace and Joshua's equations

Ms. Kelly: Thank you, for being brave and volunteering. Now, what we are going to do is we're going to have boys and girls tell you [Grace and Joshua] something really good about your sentence and then we're going to talk about a way that we can fix them up, okay, because that is how we learn. Okay, who has something nice to say about Grace's sentence? I know. Something good to say about her. I know there is some really good features there. Jessica?

Jessica: I like her first four.

Ms. Kelly: The shape of her four and her plus was, like, perfect, right? See how it is lined up right in the middle. Okay, now who can tell us something would make it better? Mm-hmm… A way to improve this? Kaylee?

Kaylee: That zero has to be smaller…

Ms. Kelly: Yeah, it's sort of lying in the air, isn't it? See, look, Grace, if we drew a straight line across it, that zero needs to be the same height and go as low as that one. Would you like to fix that one up? And then that four needs to be in line with them. Otherwise, it looks great. Okay, so why don't you fix it up with an eraser.

Grace: Okay [Revises the sentences according to the teacher's directions].

Ms. Kelly: Did she get the right answer?

Student: Yes.

Ms. Kelly: [Looking at Joshua's equation] Okay, something good about this one. Stella?

Stella: I love the way he wrote it.

Ms. Kelly: The way he wrote it? Well, what about it? Tell me what he wrote about it…

Stella: Well, it's because I like… All the numbers are the same size and they are like in perfectly straight rows.

Ms. Kelly: Perfectly straight. How could we improve it, though? Ethan?

Ethan: Make the zero a little bit bigger.
Ms. Kelly: Something else. Ashley?
Ashley: You have to put the plus sign in front of the zero.
Ms. Kelly: Ahh, you saw that. The plus sign doesn't go right under it. It is in front of the second number. So if we rewrote it, we would write four plus zero equals four [erasing and rewriting the plus sign]. But he had everything in a straight line. Remember, start our numbers at the top two. After I had written four, I would want that other number right under it. Okay? You have to be really careful. Raise your hand if you have learned something. (Classroom transcript, 04/12/11)

Regarding this lesson, Ms. Kelly spoke of its instructional purpose:

> The students are going to be seeing equations both vertically and horizontally. And I wanted them to understand that they are still the same kinds of things. So that is why we taught that lesson. So they need to be able to read vertical equations and to be able to write them also. And so that was the purpose of the lesson. So, I had the children, when I had them up to the board, one was writing them vertically and what was horizontally, I just wanted them to compare the two types. And to understand that when we put the line under the add end, that's sort of is another way of saying, "equals." So it's an equation again. (Interview transcript, 04/29/11)

Ms. Kelly thought that it was important to compare and contrast a horizontal addition equation (children's prior knowledge) with a vertical addition equation (new knowledge). To this end, she had two children write an addition problem horizontally and vertically. And then, she utilized the whole-class discussion to provide the rest of the class with opportunities to talk, listen, and share the ideas about what the good aspects were in the two children's sentences and how they could be improved. She believed that a whole-class discussion would enable them to understand and learn a new mathematical skill by comparing and contrasting the difference between horizontal equations and vertical equations effectively.

> And I had the children comment on it because sometimes I think what the children say is more powerful to the other students than when the teacher's speaking. When the teacher speaks it's sort of like when the parents speak in the Charlie Brown cartoons – where the parents are going "Waha wanha, waha wanha" and this way, sometimes students really listen to each other more than they do teachers … And plus they get to see errors and it's okay to make errors and sometimes, we try to give them compliments and then suggestions and that's sort of a social studies lesson, too – complimenting others and then how to accept criticism, too. (Interview transcript, 04/29/11)

Ms. Kelly gave students chances to participate in not only finding errors in the equations of the two children but also in suggesting how to improve them. She thought that this approach, based on a whole-class discussion format, could be a proper strategy to motivate students to actively engage in the process of developing their own mathematical understanding.

Ms. Kelly appropriately used the format of the discussion in this lesson and she tried to facilitate student talk in many ways. Nevertheless, the real dialogue described above showed that this discussion was focused on how much young children could memorize and understand of what the teacher instructed about vertical equations. Children's responses included three following aspects: the shape of the numbers, every number being in a straight line, and the position of the plus sign. All these issues are necessary skills to learn and to use in vertical equations. However, this discussion was merely to develop children's procedural understanding of how the teacher illustrated writing vertical equations (Ma, 2010). Her approach did not enable children to reach a level of conceptual understanding about vertical equations.

Ms. Kelly also failed to facilitate academically productive talk about mathematical concepts and procedures. That is, if the teacher explains the concept of place value such as one-, two-, and three-digit numerals, then children can easily understand why all the numbers should be the same size and in a straight line, instead of simply memorizing those mathematical skills. If the teacher considers the issue of why children write vertical equations in certain ways, the teacher can encourage them to discover the mathematical reasons and principles underlying vertical equations as well as to make sense of it mathematically through the classroom discussion.

To sum up, the duality of Ms. Kelly's beliefs of mathematical discussion across the contents seemed to be problematic. She assumed that there might be no way for young children to explore, discover, and discuss the meaning of a vertical equation. When teaching how to write vertical equations, she simply showed this procedural writing to students, so that they could clearly understand each step of 'how to' writing and learn how to make necessary calculations. Of course all the steps she introduced helped students, yet she failed to consider the way of writing vertical equations a problem of "know how and also know why" (Ma, 2010, p. 92). That is, she did help students improve their "procedural knowledge," to be memorized, of how to write vertical equations; she did not enable students to develop their "conceptual knowledge" to be connected with mathematical ideas, concepts, and skills (Chapin et al., 2003, p 45). She also did not provide students with opportunities to combine their understanding of place value and basic operations.

This finding extends prior research on the relationship between the teacher's beliefs about discussion and the teacher's practice in the mathematics classroom (e.g., Nathan, et al., 1997). Ms. Kelly's case is indicative of the more specific claim that the teacher's practice to facilitate mathematical discussion can be influenced by the teacher's misinformed beliefs that an effective approach differs according to content. This study also reconfirms that the teacher's carefully guided classroom discussion can be especially appropriate in developing both students' conceptual and procedural knowledge in any mathematics content (Chapin et al., 2003; O'Connell & O'Connor, 2007).

LIMITATION OF A TEACHER'S KNOWLEDGE OF CONTENT AND STUDENTS

The second challenge was the limitation of Ms. Kelly's own knowledge of mathematics content and of her students. To illustrate, I look at the following example of how discussions entered or did not enter into the instructional design.

In the middle part of a unit on numbers and operations, while half the class, a group of lower-achieving students, played mathematics games on the tables, Ms. Kelly was working with higher-achieving students at the front of the room. In this small group discussion, she told word problems and the students solved using base ten counters.

Ms. Kelly:	Okay, put them away and let's try another one. Listen carefully, and show me using the cubes. This time Bela had 18 pieces of candy, candy hearts, and David had nine candies. How many more hearts did Bela have?
Students:	[Using the cubes]
Ms. Kelly:	Can you make 18 with two sticks?

Students:	No.
Ms. Kelly:	Good. You showed me 18… Bela had 18 hearts, so put them in a line; put your 18 in a line. Put your 18 in one line…
Students:	[Putting one ten-stick and eight ones in a line.]
Ms. Kelly:	Oh! Jacob, put your 18 in one straight line… Well… Aha! Now there's another way you can do it, too. You have 18 now… Now David has nine... Aha~ Let's look at Jacob's. How did you do it? Jacob, show us.
Jacob:	I went like [See Figure 7], she [Bela] had 18, so [pointing two of the ones in the top of two ten-sticks] I just put two ones out, and then I went and covered it [pointing 9 ones in the first ten-stick], so she has 9 more [pointing 9 ones in the second ten-stick].

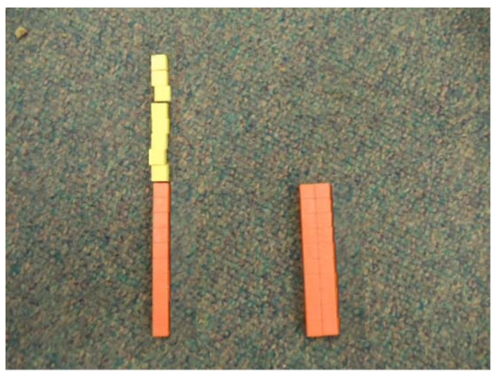

Figure 6. The students and Jacob's representation of 18 with cubes

Ms. Kelly:	Let's think about this a minute… So, you are saying that you put two ten-sticks and then covered two of the ones? So is it you think 18, right?
Jacob:	Right.
Ms. Kelly:	And David has nine. You go, "Wait, David's nine." So, one, two, three, four, five, six, seven, eight, nine. I will take those away and then how many are left beyond that?
Jacob:	Nine.
Ms. Kelly:	Exactly. Good thinking there, Jacob. So did you know you can subtract, too?
Jacob:	Um… (Class transcript, 02/24/11)

As Ms. Kelly expected, most students got out one 10-stick and eight 1's, put them in one line, and counted out eighteen. And they covered up nine, and then counted the remaining ones. On the other hand, Jacob showed a different way. He put two 10-sticks in parallel, and then he covered up two of the 1's so that the rest of them could be eighteen (see Figure 6). And he covered up nine in the first 10-stick, so that he immediately figured out Bela had nine more hearts than David. Ms. Kelly explained that at this time, she was trying to get the students to share different problem-solving strategies:

> Sometimes we have, especially teachers, we have, like, one way in our mind about how to solve a certain problem, and it's really refreshing when you see students solving it in a different way. And instead of saying, "No, you have to do it my way," we want to praise that and honor their different way of doing things. Like Jacob that used ten… He was supposed to show me 18 and normally, when I think of 18, I think of one ten-stick and eight ones. Well, he got out to two ten-sticks and he covered up two of the ones and that was 18. And I thought, "Oh, that's a good way…" So he was counting down, and that is another way of thinking about it. So we want to honor that and we want students to think of numbers in different ways because numbers are to be manipulated and used and they can be used in more than just one way or process. (Interview transcript, 03/24/11)

Ms. Kelly recognized that mathematical problem-solving strategies could be negotiated in multiple ways. Finding Jacob's idea to be different, she made a point of praising and honoring it. She reflected that this classroom discussion enabled the students to understand various ways others had of coming up with the answer. For this, she skillfully used an instructional strategy of revoicing through rephrasing Jacob's explanation to the rest of the class (O'Connor & Michaels, 1996).

Students eventually got to hear Jacob's idea. Nevertheless, Ms. Kelly's directed instruction of how to represent 18 by using cubes in the situation of a small-group discussion was likely to be limited in actively developing and expanding students' mathematical ideas. She gave students a concrete direction that 18 could not be made with two 10-sticks. She then had them put cubes of eighteen in one line. This kind of instruction seemed to be indisputable to teaching basic number facts based on each level of the digit (e.g., ones, tens, twenties, and hundreds). Thus students repeatedly practiced and learned numbers (e.g., 34 is three 10-sticks and four 1's). However, such a directed instruction should have followed a review of students' ideas or, in the end, summarizing both small points and major conclusions. In order to provide students with opportunities to share ideas through discussion, she should have first asked open-ended questions such as, "What is 18" and "How can we make eighteen with cubes?"

Furthermore, Ms. Kelly did not scaffold Jacob's understanding about the "derived number facts" (Carpenter et al., 1999, p. 24) of 18. This dialogue showed that Jacob used the strategy of doubles (e.g., 3 + 3, 8 + 8) and sums of ten (e.g., 4 + 6, 2 + 8). That is, Jacob's solution seemed to be based on understanding relations between numbers, including that 9 plus 9 equals 18, and that 9 and 1 more is 10. It is probable that Jacob's way of solving this problem was inconsistent with Ms. Kelly's understanding of how he might count cubes up to nine. According to Carpenter et al., (1999), "When children have the opportunity to discuss alternative strategies, the use of Derived Facts becomes even more prevalent" (p. 24-25). Ms. Kelly did attentively listen to and figure out Jacob's explanation. She did not, however, ask Jacob why he made 18 with two 10-sticks and why he covered two of the 1's. If she had, the discussion would have moved on to inviting the rest of the students to understand Jacob's alternative problem-solving strategy based on his understanding of doubles and sums of ten.

To sum up, the second challenge concerning Ms. Kelly's teaching practice of mathematical discussion is her own knowledge of mathematics. Previous research points out that many early childhood teachers in the United States tend to have a very narrow concept of the learning scope that children should learn, "even when children are capable of learning far more complex content" (Lee & Ginsburg, 2009, p. 39). Similarly, Ms. Kelly limited her focus on simple counting and identifying numbers in her kindergarten mathematics lessons, and, when teaching basic number facts, she favored direct instruction. She assumed that whereas mathematics problems could be solved in more than one way, the number itself could not be represented in multiple ways. She provided students no opportunity to talk about the different combinations of grouping the quantity of a certain number by tens and ones or to understand the possible relations between numbers, such as the issue of doubles and sums of ten.

Ms. Kelly's use of various approaches was appropriate to scaffold student talk in the classroom discussion. There were, however, moments when she failed to expand student talk, particularly, related to their ideas of basic number facts. She knew how to design mathematics lessons and how to use pedagogical strategies to facilitate mathematical discussion (Borko & Putnam, 1996; Shulman, 1987). Yet she did not understand or use knowledge of how 5-year-old kindergarteners "think about, know, or learn the particular content" (Hill, Ball, & Schilling, 2008, p. 375). This finding adds a perspective on the relationship between a teacher's knowledge of mathematics and a teacher's practice of facilitating classroom discussion, an issue not widely examined in primary education (Sherin, 2002). This study suggests that the ways in which early childhood teachers scaffold children's talk and learning should be supported by not only their "knowledge of content and teaching" but also their "knowledge of content and students" (Ball, Thames, & Phelps, 2008, p. 389).

TIGHT DAILY KINDERGARTEN SCHEDULE WITHIN MANDATORY STANDARDS

The third core challenge derived from the data was a tight daily kindergarten schedule within mandatory standards. As Ms. Kelly planned and implemented mathematics lessons over the year, she encountered a dilemma. On the one hand, she was trying to create a discussion-intensive mathematics classroom in which young children's contributions and interactions could be the basis of their learning. On the other hand, she should incorporate her mathematics lessons with her school district's mandatory standards and the state accountability system. This section illustrates Ms. Kelly's struggles to find a proper balance between these two forces.

Ms. Kelly noted that she felt comfortable with the state-mandated standards[7], explaining that they were basically aligned with her beliefs of classroom discussion in the

[7] Texas Education Agency (2011, p. 1-2) pointed out communication as an essential process standard for kindergarteners, by illustrating, "Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication."

process of teaching and learning mathematics (French, 2004). Also, the curriculum of her school district, Springfield Independent School District (SISD), emphasized mathematical discussion as a part of best practices to lead to each and every child's success in learning.

> I was always taught, as a teacher, as part of my pedagogical training, that you try to get kids to discover things on their own and then verbalize it. So that's what we are often trying to do. And it's a part of best practices. Our district curriculum puts out a list of, they call it a teaching and learning continuum and it has most effective practices and unacceptable practices. And the classroom discussions, that is, the children speaking to each other and engaging in learning and talking about their learning, falls into most effective practices. (Interview transcript, 03/24/11)

Ms. Kelly showed that according to "A Teaching and Learning Continuum," a curriculum document distributed by her school district, the classroom discussion was one of most effective practices for kindergartners. She agreed, and this was why she envisioned a discussion-intensive environment in which children would verbalize and exchange their mathematical ideas:

> Yes, I agree. I strongly feel that classroom discussion affects younger children's math achievement for the following reasons. Probably number one is if you can explain it and listen about it, talk about something, you just understand it better. I've had students in the past that were great at adding; they were great calculators. I mean they could go, "One plus one is two. Two plus two is three." You know, they could add. And they had strategies for adding. But when it came to concepts and explaining it and seeing how to apply it, they didn't have it. And so I want children to be able to take the math and apply it. And if you can't discuss it, you probably can't apply it … I really feel that children need to be able to talk about math. And I don't want them just to be adding. I just don't want them to be human calculators. I want them to be able to be real mathematicians and be able to see things mathematically. (Interview transcript, 04/07/11)

According to Ms. Kelly's view of mathematical discussion, the goal of teaching mathematics should be to help children become real mathematicians, to be able to explain a core concept and to apply it in their actual lives, instead of just memorizing or calculating to find the answer quickly. She addressed that if children verbalized how to solve a problem, they could understand it better and they would learn mathematics in a real way. She noted that this perspective on classroom discussions had developed with the changing mandated curriculum throughout her 33-year teaching career. Those standards were good recommendations and guidelines for her to reexamine and rebuild her own teaching beliefs and practices (Interview transcript, 05/19/11).

However, despite the benefits of a mandatory curriculum itself, Ms. Kelly's practices of mathematical discussion were actually constrained by the growing demand for academic achievement, part of the standards-based accountability in her state (Wien, 2002). The Texas Accountability System includes a set of required curriculum, called the Texas Essential Knowledge and Skills (TEKS), which are state standards for what students should know and be able to do (Texas Education Agency, 2011). In accordance with the TEKS, her school district developed the required curriculum, including the learning goals and objectives that SISD students are expected to achieve at every grade in every course. Ms. Kelly stated, "We must have the aligned SISD curriculum that, and it's a scope and a sequence, so they expect us to be teaching certain things at certain times. So that's sort of our outline, but that's also sort of strong pressure" (Interview transcript, 02/24/11). This intensification on the required knowledge and skill at the kindergarten level eventually became the powerful determinant constricting Ms. Kelly's moment-to-moment actions and interactions facilitating mathematical discussion in her kindergarten classroom. She explained that the kindergarten teachers in her school district have to teach a large amount of SISD-required mathematics content within a single academic year.

> We started really concrete. We've done a lot of the colors, the shapes, the sorting, graphing. We've worked on geometry and measurement. I have to go down my whole list of skills that we've worked on… And we started with numbers. We started at basic one and what is one, and what is one more than one, and we counted by ones to 100. And today we did that with all the numbers to 20 and are having really concrete experiences. We are always counting objects in here. And we are counting by ones and we are counting by twos. We are building number patterns. So those are the kinds of things that we have taught up to this point. And we have to come back and re-teach things. We're going to work on halves and wholes, duration and sequencing of events, and we'll work on modeling and creating addition and subtraction problems in real situation with concrete objects. (Interview transcript, 02/24/11)

Ms. Kelly illustrated what kinds of mathematics content she had covered in the first two quarters and what she would teach until the end of the spring semester. She showed that SISD presented both instructional timelines and learning standards for mathematics. (For example, in the fourth quarter, the units of halves and wholes should be taught within 5 days included 1 day for re-teach or extension as needed.) The primary focal areas in her kindergarten classroom were numbers, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. SISD also gave concrete and detailed directions on how much the teacher should teach in certain contents. She contended that these required standards pushed young children into a tight daily schedule. Hence, a teacher had insufficient time to scaffold student talk in the situation of classroom discussions.

For instance, Ms. Kelly divided the students into four small groups, and she gave students some shapes (e.g., a bus, a pie, a rectangle, a triangle, a circle, a pencil, etc.). Then, she had students discuss which one could be easily folded or cut into two equal parts, which was one of the TEKS to be mastered in the fourth quarter. After each group sat together on the carpet, Ms. Kelly went over to one small group of four students. She overheard the following interaction[8]:

Claire:	Please, pass it [a triangle] around.
Noah:	Ok. Fold it.
Ethan:	Yeah, that's equal. That's easy to do.
Noah:	That's equal. It's equal.
Claire:	Let's put it by me. So equal goes on this side . . . And then pass one [a bus] to me. This school bus is not going to be. Let me try this. Ooh… No.
Ethan:	Let me try. Maybe… You know what I can try. I can try a diagonal. I can… diagonal and then… curve this over it. And then… curve this back over and then this here and then this… here. There you go. But that was not easy. We'll put it in here. That's the not hard one… I finally figured out how to do that. But I had to fold it more than one time.
Ms. Kelly:	Did it fold equal parts, the bus?
Ethan:	Kind of, yeah.
Ms. Kelly:	Really?
Ethan:	I can show you. Let's see. I can fold it this way [on the diagonal].
Noah:	No, don't try it. I think… Okay, let me try. Bus… It's a hard one. Let's put that into the not hard…
Jessica:	You're right. It's not easy.
Claire:	Ethan, please put it in there.
Jessica:	That means not easy.
Ms. Kelly:	Okay, Ethan. Are you agreed?

8 This classroom transcript about fractions is the same one illustrated in Chapter V (pp. 75-76).

Ethan:	I think… Okay, put that in the hard pile.
Ms. Kelly:	Are you sure? Why do you think Claire and Noah want to put this into a not-easy-to-cut-into-half?
Ethan:	Because… I can try a diagonal, so I fold it into two parts, slightly equally… but I cannot fold it in half exactly.
Ms. Kelly:	So?
Ethan:	I'll put it here… Hard pile.
Jessica:	Ethan, put it in the hard pile next to me.
Noah:	Claire, you are the leader. It can be next to you. You're the leader.
Claire:	Okay…. So, who is next?
Jessica:	Is it my turn?
Claire:	Yes, it is your turn.
Jessica:	Okay, [folding a square into half] that's easy. So just put it in this [easy to fold] pile. Okay, now it's Kamala's turn. Now it's Kamala's turn.
Noah:	What does she get?
Jessica:	It's a pie plate.
Kamala:	I don't know how to fold that.
Jessica:	How about this way?
Kamala:	[Kamala folded a shape of a pie plate according to Jessica's idea.]
Ms. Kelly:	Oh, does it work that way, Kamala?
Kamala:	Yes, kind of…
Ms. Kelly:	Is it two equal parts?
Kamala:	Yes.
Ms. Kelly:	Is there another way?
Kamala:	No.
Jessica:	We can put it in that [easy]) pile.
Ms. Kelly:	Is there another way she [Kamala]) could fold it?
Ethan:	Uh, no way.
Ms. Kelly:	Ooh.
Ethan:	So put it in the easy pile. Okay, that was easy. Okay, now it's my turn.
Claire:	It's your turn for the string.
Ethan:	That's easy.
Jessica:	Yes.
Ethan:	Yep, that's easy to fold into two.
Claire:	So put it in this [easy] pile.
Noah:	Okay. That's easy. Put it in there.
Claire:	So put it in that pile. We're done. (Class transcript, 03/22/11)

This dialogue shows how Ms. Kelly tried to attentively listen and appropriately scaffold student talk in the learning situation of a small group discussion. She caught the moment that Claire and Ethan had different ideas of how to fold the shape of the school bus into equal halves. She asked Ethan a question about verbalizing his idea clearly. This was so he could reexamine his idea and the rest of the group could have a chance to review their ideas. She also encouraged Kamala, reluctant to speak aloud, to participate safely in a small group discussion. She did this by giving a supportive response to what she was doing. She was aware of her role as a facilitator in small group discussion: within a limited time of 5 to 20 minutes, she was very busy quickly roaming the classroom, stopping at each small group to scaffold as much as possible.

Nevertheless, as Ms. Kelly reflected on this lesson at the second interview (03/24/11), she lamented how parts of it went. She said it was unfortunate that, due to the tight daily schedule, she could not spend more of the discussion starting from Ethan and that she did not support passive and calm students like Kamala to engage much more in the interaction of a small group discussion. She said, "we do the best we can, but there just isn't enough time in the day to do it" (Interview transcript, 03/24/11). Although on that day, the mathematics lesson fell on the second day of a unit on fractions focusing on halves and wholes, the SISD's present kindergarten standards forced the pace of her instruction. This was because she had to cover all the mathematics content and all the teaching materials on how to share a whole. This included by separating it into two equal parts and why a given part was called one-half of the whole. This was what SISD required kindergarten teachers to do within five days during the academic year's fourth quarter. This specific timeline and the required contents allowed the teacher to regularly plan and designate the mathematics lessons, yet, conversely, it forced her to lose the autonomy of teaching these mandated knowledge and skills according to her pedagogical beliefs concerning classroom discussions (Goldstein, 2007).

For instance, she continued to explain how the tight daily schedule caused by SISD mandatory curriculum affected her daily moment-to-moment decisions. Ms. Kelly initially assumed that Ethan's idea to be wrong because the shape of the bus could not easily be folded by hand into equal parts. However, she realized that his idea was theoretically right based on the definition of half of something—one of two equal parts that together make up the whole number, amount, or object. She guessed that Ethan, one of her excellent students, might think the gross area of the car shape itself could be divided into two equal parts by cutting it repeatedly, probably more than one time as Ethan said. After hearing Ethan's idea, she became aware that how she had expressed how to easily fold or cut into two equal parts was problematic for explaining the concept of half for 5-year kindergarteners. As a result, her approach to explaining a half might confuse students into thinking they could fold or cut a shape into two equal parts only when the shape was symmetrical (e.g., a triangle, a rectangle, a pie, etc.). Although Ethan's idea was great, he revised it and eventually accepted the three other students' opinion that the car shape could not be easily folded into two parts. She sadly pointed out that, at that moment, she should have expanded Ethan's idea for the rest of the class to rethink about the definition of half. She didn't, though, because due to the full agenda there was not enough time to have this kind of the discussion.

To sum up, the third challenge in determining how Ms. Kelly facilitated mathematical discussion is a tight daily kindergarten schedule within mandatory standards. Previous research has pointed out that the dilemmas and challenges a teacher faces in increasing discussions in mathematical classrooms can be derived from three core things: the teacher's experiences with more traditional tasks and pedagogy, the teacher's subject matter knowledge, and the capacity of colleagues to be supportive (Silver & Smith, 1996). On the other hand, this study adds the perspective that the growing demands for academic pursuits around Ms. Kelly's kindergarten classroom, constrained by her state and school district mandatory curriculum (Wien, 2002, 2004), could be a critical factor to her effort to scaffold student talk and learning. She as a public kindergarten teacher was under obligation to cover a large amount of the mathematics content within a single academic year. It pushed kindergartners into a tight daily schedule, and also forced the pace of her instructions for scaffolding student talk.

PARENTAL EXPECTATIONS INDUCED BY PRESSURES OF HIGH-STAKES STANDARDIZED TESTING

The fourth challenge was the parental expectations induced by pressures of high-stakes standardized testing. Ms. Kelly pointed out that the state's high-stakes standardized test also constrained her teaching practices to enact a discussion-intensive kindergarten classroom. The Texas Accountability System integrates the state criterion-referenced achievement test, which is called as the Texas Assessment of Knowledge and Skills (TAKS) to assess public school students on what they have learned in certain subject areas and to determine district and school accountability ratings (Texas Education Agency, n.d.b). Ms. Kelly claimed that although this statewide assessment started in third grade, it had turned the kindergarten into the first competing stage. It also had compelled the kindergarten teacher teams in her school district to plan and implement mathematical lessons as essential prerequisites for young children's success in the near future. Within this climate, Ms. Kelly, one of several senior teachers having over 30 years of teaching experience, was granted instructional freedom by her principal. She did not experience the first grade or lower-grade teacher's demands. These teachers usually wanted kindergarten teachers to focus more directly on instructing academics skills. Goldstein (2007) found the pressure from first grade teachers as a major challenge facing kindergarten teachers. However, the pressure from her students' parents was a significant factor in her planning and implementing of mathematics lessons.

Ms. Kelly felt a certain obligation to satisfy her students' parents. They expected their children to achieve excellent TAKS scores three year down the road. She described that her elementary school was situated in an affluent suburban neighborhood, and the majority of the student population consisted of Whites and Asians. Most of their parents had relatively high-level education backgrounds and they were deeply concerned about their children's academic achievements. Also, her elementary school received an "exemplary" accountability rating[9] for the 2010-2011 school year and earned high

[9] An "exemplary" accountability rating means a passing rate of 90% of the 3rd, 4th, and 5th-grade students for each subject (Texas Education Agency, n.d.a)

performance acknowledgement[10] in reading, writing, science, and mathematics. Parents thus not only expected their children to completely master the mandatory essential knowledge and skills in each subject but also wanted them prepared to score high on the standardized tests. Ms. Kelly often encountered such expectations, expectations that actually conflicted with hers, of children's academic skill development in mathematics.

> Over the years I have accumulated many, many lessons, and we get to provide the lessons. We do not use the adopted, um, textbook because we do not like the papers that are included in it. So we just ask not to even receive it. And it didn't have very many manipulatives. So we're always using lots of objects in here, lots of objects. I want them to be able to see the things. I want math to be at the application level… Keep things concrete; use manipulatives; use a variety of activities; get them to speak – that always helps... I don't want students to remember that three times five is fifteen. I want them to be able to explain, "Oh, if I have three groups of five, that equals fifteen." Because you know, if they can verbalize it, then they really know it. Otherwise, if they just doing things by rote and they never have to reflect upon it, they really don't know it as well… But parents want students to be taking it to the paper, too, because they're going to have to be recording things, recording data, and taking tests, and stuff. So they want them to have the paper and pencil activities as well. (Interview transcript, 02/24/11)

Ms. Kelly's kindergarten mathematics lessons, rather than coming from the adopted textbook, were developed and accumulated from her long teaching experience. She pointed out that her teaching purpose in mathematics was focused on developing young children's understanding of mathematics at the application level related to the real world. For this, she argued that the teacher should provide children with many opportunities to manipulate lots of objects, engage in a variety of activities, and to verbalize their ideas. In

10 The Gold Performance Acknowledgment (GPA) system acknowledges districts and campuses for high performance on indicators other than those used to determine accountability ratings. (Texas Education Agency, n.d.a)

contrast, her students' parents wanted their children to be taking mathematics to paper and pencil so as to be familiar with taking tests even in kindergarten contexts. This pressure she was experiencing thus pushed her into using more direct instruction for a greater focus on academic skills.

These differences between the positions of Ms. Kelly and her students' parents caused her to experience difficulty in acting on her own beliefs regarding the benefits of discussions in young children's mathematics learning and development. For example, she needed to decide how to plan and implement mathematics lessons in conjunction with the parents' preferred approaches to focusing on children's academic skills, such as using worksheets that are regarded as more traditional materials.

> After doing some activities, many parents think that we must have a worksheet because kids have to work on paper eventually. They will be tested on paper. So we could teach math just fine without worksheets, but parents still want students to be tested on it and they're going to have to show you [parents] on paper. So that's why we do worksheets, too. I try to I scaffold student learning as much as I did through actions, through manipulatives, or through discussions, and then also to do it on paper. (Interview transcript, 02/24/11)

To satisfy these parents, Ms. Kelly used traditional materials. However, she did not utilize such worksheets in the traditional teacher-centered methods focused on rote learning and memorization. She first tried to scaffold student learning through actual doing, manipulating, and discussing, and then she had students do worksheets. It was another way for students to see their understandings of what they had previously discussed and learned. Ms. Kelly also elaborated on how to manage the expectations of her students' parents.

> We communicate primarily through the Tuesday folder. Everyone in our school takes home their work in a Tuesday folder and sometimes we put in homework – often not, in kindergarten. And I keep a lot of their math work in a math composition book, so I can have sort of a history throughout the year of where they were and how they are doing. And when I am checking papers, if, for instance, Matthew was having difficulty using number lines or something, so I wrote on his paper, "Please, have him practice the number lines." And I attached the number line and he went home and – after I had re-taught it to him again – they need to get home and practice again. And he came back and said, "Now I understand it." And that, so we communicate through, lots of times, many times, just little quick notes on the work that goes home… And then I also use the composition books in parent conferences and show them my assessment of what I need to teach and what I need to re-teach, so that they're understanding the processes of children's learning. (Interview transcript, 04/29/11)

Ms. Kelly used student folders to communicate with parents about their children's academic skill development in mathematics. She had students take home their work folders every week so that parents would know what students had learned during the week. Through this weekly folder, she tried to give parents a detailed description of which parts students had difficulty learning and the specific request of how to review the questions. By documenting students' works throughout a year, parents could figure out how much their children's learning improved. She also used parent conferences and showed other evidence that she collected through observing, assessing, and documenting what their children had learned and which parts they should relearn. As Powell (1998) pointed out, Ms. Kelly believed that friendly and continual communication could provide excellent opportunities to share, learn, and be aware of the ideas and opinions of one another. All her efforts to satisfy parents regarding students' academic achievements indicated that while she held strong pedagogical beliefs about the role of discussions in student learning, she found and used the best way to integrate the parents' preferred approaches into her mathematics lessons.

To sum up, Ms. Kelly struggled to reconcile her teaching practice of mathematical discussion with the parental expectation of their young children's desired outcomes based on state-legislated accountability. She felt a pressure from her students' parents who wanted the kindergarten teacher to enable their children not only to completely master the state mandatory standards in each subject, but also to be ready to get high-scores in the state criterion-referenced achievement test. Goldstein (1998) and Wien (2004) found that many kindergarten parents tended to be uninterested in play-based or child-centered learning, which would serve as the best practices for young children most early childhood teachers had learned and believed in. This difference between early childhood teachers and their students' parents in terms of preferred approaches to teaching young children has been aggravated by the advent of standards-based education (Goldstein, 2007). It was found that a critical factor influencing not only teachers' ability to establish collaborative partnerships with parents but also their teaching practices (Powell, 1998). Similarly, this study found that the tension between Ms. Kelly and her students' parents caused by their different preferences of educational goals and pedagogical approaches in teaching mathematics challenged her pedagogical strategies to engage young children in meaningful discussions.

Ms. Kelly, an experienced teacher, was clearly aware of her own struggles that came from without, that is, constrained by both state mandatory curriculum and standardized assessments. She tried to compromise her instructional strategies for creating discussion-intensive mathematics classrooms under a standards-based accountability system (Jung & Reifel, 2011). Her case raises the issue of how to help early childhood inservice teachers recognize their own struggles and facilitate connections between their beliefs regarding mathematical discussion and the complexity of the current public school system. This study provides early childhood teachers and teacher educators with implications on how to deal with the new recommended practices and mandatory standards and the growing demands for academic pursuits within the field of early childhood education.

TEACHER'S ROLE IN MATHEMATICAL DISCUSSION WITHIN CHALLENGES

Ms. Kelly certainly recognized the importance of creating a respectful learning atmosphere for students' engagement in mathematical discussion. She was also able to utilize pedagogical strategies to scaffold student talks effectively. And yet, her conceptions and practices of mathematical discussion were made more difficult by the challenges discussed throughout this chapter. Again, these consisted of (1) *the duality of a teacher's beliefs of discussion depending on mathematics content*, (2) *the limitation of a teacher's knowledge of content and students,* (3) *a tight daily kindergarten schedule within mandatory standards,* and (4) *parental expectations induced by the pressures of high-stakes standardized testing*. The findings showed that a teacher's moment-by-moment decision making on how to organize, initiate, and facilitate mathematical discussion depended very much on what the teacher understood about mathematics teaching and learning, as well as how the teacher was able to deal with the constraints of mandatory standards and an accountability system.

In Figure 7, I illustrate a diagram[11] representing a teacher's role in mathematical discussion within those challenges.

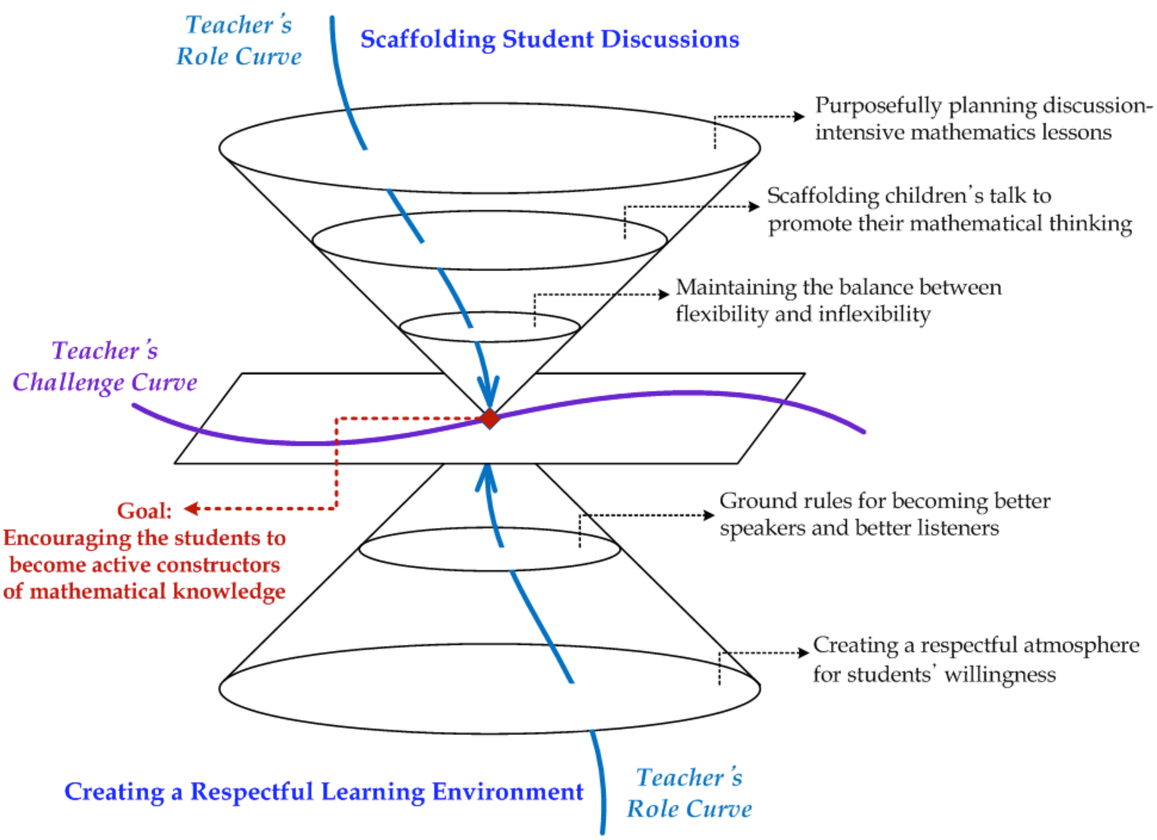

Figure 7. One kindergarten teacher's role in mathematical discussion

[11] I developed this diagram inspired from *Minkowski spacetime**
*Resources: http://en.wikipedia.org/wiki/Minkowski_space;
http://www.pitt.edu/~jdnorton/teaching/HPS_0410/chapters/spacetime/index.html

As this diagram illustrates, two kinds of cones represent the interdependent relationship between the two core roles of one teacher for initiating and facilitating a discussion-rich classroom in her kindergarten classroom: (1) creating a respectful learning environment and (2) scaffolding student discussions. The two cones' vertexes head towards the ultimate goal of mathematical discussion that children become active knowledge constructors in their own learning. The plane, which intersects these two cones, represents the dynamics of enacting mathematical discussion. The purple curve represents challenges the teacher faces in doing so. It shows that the goal of mathematical discussion can be accomplished by the teacher's two roles for promoting children's willingness to participate in mathematical discussion and for developing their mathematical thinking and reasoning; its success and failure can also depend on the teacher's ability to overcome challenges she faces in integrating mathematical discussion into her everyday lessons. Based on the findings that this diagram illustrates, I discuss their implications, the limitations of this study, and further study in the next chapter.

Chapter VIII. Conclusions

In this concluding chapter, I first present a brief summary of the findings of one kindergarten teacher's role in mathematical discussion, discussed in Chapters 5-7. Then I discuss the implications of these findings for teachers, for teacher educators, and for researchers connected to the field of early childhood educational field. I finally demonstrate the limitations of the study and make recommendations for further research in this area.

SUMMARY OF THE FINDINGS

For the purpose of looking closely at how one kindergarten teacher tried to help young children have more mathematical discussions, the research was guided by the following three questions.

1. How does a kindergarten teacher conceptualize his/her role in mathematical discussion for young children?
2. How does a kindergarten teacher orchestrate young children's participation in mathematical discussion during mathematics lessons?

Through this four-month-long qualitative case study, I found three major roles of Ms. Kelly for intensive-discussion mathematic lessons: (1) *creating a respectful learning environment*, (2) *scaffolding student discussions,* and (3) *overcoming challenges to mathematical discussion*.

Creating a respectful learning environment

The primary role of a teacher in getting started with mathematical discussion for young children is to establish a respectful learning environment so as to motivate their willingness to participate in classroom discussions. I made a series of arguments focusing on the following five aspects: (1) *creating respectful atmosphere for promoting participants' willingness,* (2) *motivating emotions in classroom discussion,* (3) *initiating ground rules for becoming better speakers and better listeners,* (4) *respectful talk for a discussion-rich classroom,* and (5) *equitable participation in classroom discussion*.

First, Ms. Kelly emphasized that the teacher should play a key role in building a respectful atmosphere to promote young children' willingness to participate in mathematical discussion. According to her understanding, if they are comfortable expressing their illogical ideas as well as their accurate answers, they are willing to explore deeply the content of mathematics. Such an atmosphere also can allow that their contributions are valued and listened to and that they feel comfortable about voicing their thoughts in mathematical discussions.

Second, in Ms. Kelly's view, students inevitably, when learning any subject matter, encounter various kinds of emotion (e.g., excitement, enjoyment, anxiety, and boredom). It is in such moments, the teacher must appropriately motivate students' emotion to stimulate their deep involvement and persistence in mathematical discussion. She knew that just focusing on mathematics cognitively was insufficient for leading students to explore the content of mathematics effectively. She recognized that students' emotional reaction in learning mathematics could interact with their cognitive and motivational processes as they took part in classroom discussions. In this respect, she emphasized her role in scaffolding students' emotional responses.

Third, Ms. Kelly further positioned herself as both an initiator and a maintainer of ground rules for becoming better speakers and better listeners. She pointed out that by explaining what she expected from students during mathematics discussion, she began to establish social norms for young children. Then, to get them accustomed to the norms of courteous discussion, Ms. Kelly provided students opportunities, over the course of the school year, to be reminded of how to talk and listen. Ms. Kelly also asserted that, after introducing her expectations of discussions, the teacher should invite young children to participate in building classroom norms. She assumed that students' involvement in the decision-making process of ground rules for discussion helped secure their intellectual ownership of learning, as well as motivated them to engage in mathematical discussion.

Fourth, Ms. Kelly argued that, to become good listeners and good speakers, young children should develop specific skills for respectful talk to politely manage face-to-face interactions in mathematical discussion. For this, she first explained to them their relational rights and responsibilities in a classroom discussion. On the one hand, each student had both a right to say his/her ideas comfortably and a responsibility to listen, attentively, to what others say. On the other hand, each student had a right to listen to others' ideas so as to convince or to expand his/her own ideas, as well as a responsibility to express his/her ideas to the others so as to clarify and shore up his/her own understanding. She also emphasized each student's obligation to be respectful of each other in the classroom. She thought it important to teach young children courtesy and proper behavior at such a young age. She thus asserted that this kind of teacher-initiated discussion could gradually improve young children's engagement in classroom discussions.

Last, Ms. Kelly emphasized her role in creating ground rules for promoting the equitable participation of every student in mathematical discussion. She used several strategies to ensure that all students, at least once every few lessons, had the opportunity to contribute to classroom discussion. Her first strategy was, in the whole group discussion, asking easier mathematics questions to low achievement students. She explained that lower level questions that fell within the range of struggling students' actual levels could motivate them to participate in discussion. A second strategy was assigning turns, with hands raised or not, by calling on students. Such a strategy was more effective, she believed, for silent students reluctant to voice their ideas and for passive students reluctant to raise their hands during mathematics lessons. Moreover, Ms. Kelly believed partner talk was also effective with such students. She insisted that students could have more opportunities to verbalize their ideas in discussion with pairs, rather than large group discussions or other types of small group discussions, even for a short time, one to five minutes.

Scaffolding student discussions

The second role of the teacher is scaffolding student discussions. According to Ms. Kelly, a respectful learning atmosphere, while necessary to enable young children to participate in mathematical discussion, was not sufficient for making students active meaning makers as they constructed their own mathematical knowledge. To scaffold young children's talk and learning, her mathematical discussions relied on (1) *purposefully planning discussion-intensive mathematics lessons*, (2) *scaffolding student talk to improve their mathematical thinking*, and (3) *maintaining the balance between flexibility and inflexibility*.

As the first category of her instructional strategies to facilitate mathematical discussion, purposefully planning discussion-intensive mathematical lessons involved Ms. Kelly's carefully selecting cognitively demanding tasks and talk formats. She did this so that students' participation in mathematical discussion would enhance their mathematical thinking and content knowledge. She indicated that the success and failure of the discussion highly depended on mathematical tasks. For mathematical discussion, she used cognitively demanding tasks that could be solved in multiple ways. Such tasks invigorated discussions. She also believed that students could be challenged and excited if the tasks included what they had already learned and experienced. She tried to offer students mathematical tasks that would be cognitively challenging as well as connected to their daily lives and prior knowledge.

Ms. Kelly further emphasized the role of productive talk. In each mathematics lesson, she used two or three talk formats such as whole-class discussion, small group discussion, and partner talk. Each talk format had strengths and limitations. She recognized that whole-class discussions could allow students to generate different ideas in the opening part of a lesson, as well as to compare and contrast those ideas and to narrow the focus for reaching the core mathematics contents in the closing part of the lesson. She also employed two types of small-group discussions. She indicated that the teacher-guided small group discussions enabled both low-achieving students and high-achieving students more opportunities to verbalize their ideas at their achievement levels. The other type of small group discussion focused on conversations between students. This type of small group discussion with three to seven students was to give more time for them to talk and share their ideas. In such manner, students became more independent thinkers as they explored mathematical tasks. Moreover, she used partner talk in order to maximize opportunities for everyone, including silent students and English language learners, to talk aloud within the limited time of a mathematics lesson.

The second category of strategies that Ms. Kelly used in pedagogical decision-making was scaffolding students' speech to promote their thinking. Ms. Kelly asserted that as a facilitator she should help students improve their thinking through appropriate questions and responses in the interactive process of mathematical discussion. For this, she used four major strategies. First, she indicated that the teacher, instead of directly forcing the right answer on the student, should use *how* and *why* questions to help students' ideas become more visible. She asserted that probing students' answers helped students recall essential facts and rethink their ideas deeply. Second, she stressed revoicing (O'Connor & Michaels, 1996), especially for kindergarteners, who must struggle to verbalize their ideas clearly. She asserted that the teacher's revoicing, through repeating and rephrasing students' explanations, enabled the speaker to clarify his/her ideas, as well as the rest of the class to understand how the discussion proceeded.

Ms. Kelly also recognized the importance of wait time for facilitating mathematical discussion. She consciously and deliberately used wait time so that some students could rethink their ideas before they spoke aloud. It also provided reticent students extra time to find a solution before they heard others' explanations. As for this last strategy for scaffolding students' talks, Ms. Kelly was aware of the important role of stepping in and out of mathematical discussion (Rittenhouse, 1998). On the one hand, she was a participant in a discussion attentively listened to students' explanations and carefully responded to their ideas in order to enable students to become more conversational. On the other hand, she played a role as the commentator by posing appropriate questions and coordinating students' different ideas, so that students could reach the focal points of mathematics content through discussions.

The third category of Ms. Kelly's strategies reflected the need, in mathematical discussion, to balance flexibility and inflexibility. In the stage of planning the mathematics lessons, Ms. Kelly tried to predict how to implement the lesson through anticipating as many student responses and points of confusion as possible. However, in the midst of lessons, she often changed her planned lessons if she came up against unexpected replies. To facilitate classroom discussions, there was a need for improvisational performance. The episode involving the bean seeds shows how Ms. Kelly shifted gears between flexibility and inflexibility. Even when students responded out of the range she predicted, she was interactive and sensitive to stimulate students to verbalize their ideas. She chose to modify the plan to get students to participate more actively in the discussion. However, she did not get so lost as to have students unable to achieve the goals of her planned lessons. While she was flexible and supportive of students' unexpected replies, sooner or later she directed the discussion toward the planned paths. At the end of the discussion, she summarized and reviewed what students had discussed, so that students were sure to have been directed to the focal points of the discussion.

Overcoming challenges to mathematical discussion

The third role of the teacher is overcoming challenges to mathematical discussion. As Ms. Kelly planned and facilitated the classroom discussion as the core way of teaching mathematics in her kindergarten classroom, she encountered four major challenges: (1) *the duality of a teacher's beliefs of discussion depending on mathematics content*, (2) *the limitation of a teacher's knowledge of content and of her students*, (3) *a tight daily kindergarten schedule within mandatory standards,* and (4) *parental expectations induced by pressures of high-stakes standardized testing*.

The first challenge that Ms. Kelly needed to overcome for discussion-intensive mathematical lessons was her own beliefs concerning mathematical discussion. Despite her emphasis on the role of classroom discussion in children's learning, this study found that how she used them depended on mathematics content. Whereas she basically tried to utilize various formats of classroom discussions, she tended to choose direct instruction when teaching basic number facts and operations. For instance, at the end of the unit on addition, she directly instructed how to write vertical addition equations. She assumed that discussion-centered approaches would not be effective at introducing a new mathematical skill, and various ideas derived from the discussions confused students regarding its specific procedures. In this respect, her instruction focused only on how to write vertical equations rather than on why to write them in that manner. On the other hand, in the activity that followed introducing vertical equations, she utilized a whole-class discussion for students to compare and contrast the differences between horizontal and vertical equations. She used the instructional strategies to encourage students to talk, listen, and share their ideas. The problem, however, was that those strategies failed to help students reach a level of conceptual understanding about vertical equations. The discussion she facilitated only enabled students to review what they had memorized about writing vertical equations. The students did not, however, understand the relationship between place value and basic operations or explore the mathematical reasons underlying vertical equations, such as why all the numbers should be the same size or in a straight line.

The second challenge was the limitation of Ms. Kelly's own knowledge of mathematics content and of her students. When teaching basic number facts, Ms. Kelly's use of mathematical discussion had a certain limitation in expanding students' ideas and in building connections among ideas. In the small-group discussion on the lesson concerning numbers and operations, she carefully observed what students' were doing. She appropriately responded to students' ideas of how to solve a different unknown problem. (Bela had 19 pieces of candy, and David had 9. How many more pieces did Bela have?) She encouraged students to share different ways of problem-solving strategies. Although her use of various approaches was appropriate to scaffold student talk, two moments arose in that episode when she failed to expand student talk. First, the manner in which she directed students to use cubes to represent 18 (18 could not make with two ten sticks; one 10-stick and eight cubes had to be put in one line) prevented them from sharing, through discussion, more ideas. Such instructions were more appropriate after reviewing students' ideas or in order to summarize both small points and major conclusions. Second, she used the strategy of revoicing in order for the rest of the group members to have an opportunity to listen to Jacob's unique idea. She didn't catch that Jacob's solution was based on his understanding of relations between numbers (9 plus 9 equals 18, and 9 plus 1 more is 10). If she had asked him why he made 18 with two 10-sticks and then covering two of the 1's, she would have moved the discussion forward on the issue of doubles and sums of ten.

The third core challenge derived from the data is a tight daily kindergarten schedule within mandatory standards. Although Ms. Kelly tried to utilize instructional strategies to scaffold student talk in mathematics lessons, her beliefs and practices of mathematical discussions had to be reconciled with the constraints posed by the Texas Accountability system. Basically, her school district curriculum and the state curricular standards were aligned with her beliefs of classroom discussion in the process of student mathematics learning. One of the curriculum documents her school district distributed, for example, regarded classroom discussion as one of the most effective practices, and also argued that the teacher should create a discussion-intensive classroom environment even for kindergarteners. She also pointed out that these mandatory standards of her school district and state were good guidelines for her to refer to when reexamining, rebuilding, and improving her own mathematics lessons. Nevertheless, she needed to find a way to integrate her teaching practices of mathematical discussion into a large amount of the mathematics content and the teaching materials that her school district required taught within a single academic year. She argued that due to these mandatory standards, young children were pushed into a tight daily schedule. She did not have as much time to scaffold student talk as she would have liked. The required contents within the specific timeline helped her designate the mathematics lessons everyday regularly, lining them up with the scope and sequence of standardized curriculum. However, she lost her instructional autonomy of how to teach, within her professional beliefs, those mandatory skills and knowledge, beliefs borne from her extensive teaching experience.

The fourth challenge is the parental expectations induced by the pressures of high-stakes standardized testing. Ms. Kelly indicated that her effort to scaffold student talk through classroom discussion was also constrained by the state's high-stakes standardized test, though it would not be administered to her students until they reached the third grade. Granted instructional independence by her principal, she was under little pressure from first grade teachers or other lower-grade teachers. Teachers from these grades usually wanted kindergarten teachers to focus more on directly instructing academics skills. On the other hand, she strongly felt a certain obligation to satisfy the parents' expectations, though they conflicted, occasionally, with hers. Parents expected the kindergarten teacher to enable their children not only to completely master the state mandatory standards in each subject, but also to be ready to get high-scores in the state criterion-referenced achievement test. Parents demanded the kindergarten teacher use more direct instruction for a greater focus on academic skills; they demanded to see visual evidence of their children's math performance. To reconcile these expectations with her own beliefs about best practices for young children's mathematics learning, Ms. Kelly decided to use such traditional materials in a non-traditional way. For instance, she first tried to scaffold student talk and learning through actual doing, manipulating, and discussing. She then, at the end of the lesson, had students do worksheets to review their understanding of what they had discussed and learned. Communicating with parents through student folders and parent-teacher conferences, she filled parents in on what the students had learned, how much they had improved, and where they currently were. These efforts showed Ms. Kelly amenable to actively integrating parents' preferred approaches into her discussion-intensive mathematics lessons.

IMPLICATIONS

This study's findings offer significant implications for those who want to enhance the development of young children's mathematical competencies, that is, for early childhood teachers, educators, and administrators.

For practice: Emotional and cognitive scaffolding for children's talk

In getting started with classroom discussions, early childhood teachers should be aware of emotional scaffolding (Rosiek, 2006). Emotional scaffolding mediates the formation of children's mathematical identities, dispositions, and confidence in learning. Previous research has continuously indicated the importance of a classroom environment where students feel comfortable enough to ask questions and make mistakes (Ball & Friel, 1991; Chapin et al., 2003; Cobb et al., 1998; O'Connell & O'Conner, 2007; Vacc, 1993). According to Chapin et al., (2003), the procedure of mathematical discussion includes not only cognitive aspects but also social aspects of student talk in mathematics learning. They resist the notion that both "creating productive talk about the actual content of mathematics" and "establish[ing] a supportive learning environment" (p. 6) are of equal importance. On the other hand, many early childhood teachers often limit their focus to providing an enriched physical environment with a variety of mathematical objects and materials (Lee & Ginsburg, 2009) or to designating the physical arrangement of the classroom that can help building the optimal environment for communication between students (O'Connell & O'Connor, 2003; Vacc, 1993) According to Lee and Ginsburg (2009), "[a] rich physical environment, while an important indicator of quality, is not enough by itself" (p. 40).

This study shows how an experienced kindergarten teacher was able to successfully establish such a respectful learning environment. Her classroom environment emotionally scaffolded young children's willingness to participate in mathematical discussion. It provides a detailed analysis of how a teacher laid down ground rules for supportive and courteous talk in her kindergarten classroom. It displays how she maintained those norms to make her students better speakers and better listeners across the academic year. It also shows how a teacher can encourage young children to be familiar with the ways of managing face-to-face interactions and to feel comfortable participating in talking and sharing their ideas and mistakes. The presented examples and descriptions in this study offers significant implications for early childhood teachers who truly care about their young students' mathematical development, and who work hard at developing trusting classroom communities but unsure of how to initiate mathematical discussion.

The teachers also should be exposed to and become familiar with a range of instructional strategies, which, through the process of mathematical discussion, cognitively scaffold young children's talk and learning. An abundance of literature on mathematical discussion is available (e.g., Chapin et al., 2003; Empson, 2003; O'Connell & O'Connor, 2007; O'Connor & Michaels, 1996; Pierson et al., 2007; Walshaw & Anthony, 2008; Wood et al., 1993). Nevertheless, many early childhood teachers are relatively unfamiliar with classroom discussion and lack experience with how to support and encourage young children to talk, listen, and share their ideas (Pierson et al., 2007; Rudd et al., 2008; Schwartz & Brown, 1995; Whitin & Whitin, 2003). This may be due to three reasons. First, according to Lee and Ginsburg (2009), many teachers may have a misconception that "language and literacy are by far the most important topics to be taught in early childhood, and that a focus on these subjects leaves little time for mathematics" (p. 40). Second, as Hammerness et al., (2005) point out, teachers may prefer teacher-directed approaches, a preference that grew out of their own experiences with these highly structured ways. Finally, Skipper and Collins (2003) assert that teachers' lack of understanding about a child-centered, play-based approach may have led to another common misconception, such as children's mathematical learning occurs incidentally only through touching and moving concrete objects during free play, with little teacher participation (Lee & Ginsburg, 2009).

However, the participant teacher in this study believed that young children's understanding of mathematics takes place most effectively in a discussion-intensive learning context. Here they can explain, justify, and share their ideas with their peers. This study demonstrates the explicit and practical ways of how a kindergarten teacher can scaffold young children's talk and learning in her mathematics lessons. It illustrates specific pedagogical strategies for purposefully planning discussion-intensive mathematics lessons, for scaffolding student talk to improve their mathematical thinking, and for maintaining a balance between flexibility and inflexibility in the midst of the discussion. These strategies are basically aligned with approaches many previous studies have continually suggested (e.g., Chapin et al., 2003; O'Connor & Michaels, 1996). They have, however, been revised, modified, and developed by one experienced kindergarten teacher. She did so to facilitate, in a way of developmentally appropriate practice (DAP), mathematical discussion in her everyday mathematics lessons. The DAP refers to professional knowledge of age-related characteristics and thus promotes young children's optimal learning and development in mathematics (Copple & Bredekamp, 2009). This study will thus be a source of detailed practical information for in-service teachers in early childhood educational field.

Furthermore, this study provides implications for early childhood teachers who must reconcile their teaching practices with the constraints imposed by the public school system. According to Engel's (2011) observations, many young students "spend their days identifying letters, reciting written words, answering specific kinds of questions, and enacting routines" (p. 636), and many teachers "feel compelled to make sure children learn what is included on standardized tests" (p. 636). As a result, Engel (2011) pointed out that mastering specific skills and knowledge for academic goals at each grade level, rather than inquiry into the content by following their curiosity, seemed to be the dominant goal for almost all classrooms. Furthermore, this constraint within mandatory standard and high-stakes standardized test have led teachers to underemphasize classroom discussion, going against the views of socio-constructivists, who believed the discussion was one of the best socially mediated contexts for young children's mathematics learning (Jung & Reifel, 2011).

The current study also reveals that mandatory standards have pushed young children into a tight daily schedule. The participant teacher, as a result, had not as much time to scaffold student talk, as she would have liked. Students' parents, moreover, pressured her with their expectations of their children being ready to attain high-scores in the state criterion-referenced achievement test. In the face of such challenges, Ms. Kelly was able to hold to her own beliefs of classroom discussions in teaching and learning mathematics within a standard-based accountability system. She facilitated a discussion-rich classroom as an essential way of not only integrating the mandated curriculum into kindergarten mathematics lessons, but also of satisfying parental expectations. This case study of one experienced kindergarten teacher is a source for early childhood teachers on how to deal with the constraints of mandatory standards and an accountability system as well as on how to integrate mathematical discussion into their everyday mathematics lessons.

For teacher educators: Teacher knowledge and decision-making

The current study also provides insights into how teacher educators can help early childhood preservice teachers develop a profound understanding of mathematics teaching and learning. Equipped with such an understanding highly influences their moment-by-moment decision making to appropriately scaffold young children's talk and thus to develop a mathematically grounded understanding. Research, theoretical and empirical, has continually been conducted on the mathematical knowledge needed for teaching (e.g., Ball et al, 2008; Hill et al., 2008a; Lampert, 2001; Ma, 1999, 2010; Shulman, 1987). The research has correlated teachers' knowledge with student achievement in mathematics (Hill, Rowan, & Ball, 2005). It is also associated with the mathematical quality of instruction (Hill et al., 2008b).

Still, few early childhood teachers have, according to Ginsburg and Ertle (2008), been trained to teach mathematics to young children. Even if they had, their knowledge would not be deep because "organized and rigorous mathematics curricula is a rarity in early childhood education" (p. 46). The finding in this study draws attention to the fact that the participant teacher, capable of creating a respectful learning environment and utilizing pedagogical strategies to foster young children's talk, did not necessarily always produce the desired effect. This study revealed that she could stand to develop her "knowledge of content and teaching," which "combines knowing about teaching and knowing about mathematics" (Ball et al., 2008, p. 401). And yet, her mathematical "knowledge of content and students," which "combines knowing about students and knowing about mathematics" (Ball et al., 2008, p. 401), was insufficient to develop both students' conceptual and procedural knowledge of any mathematics content.

This shows that a teacher's moment-by-moment decision making on how to organize, initiate, and facilitate mathematical discussion depends very much on what the teacher understands about mathematics teaching and learning (Anthony & Walshaw, 2009). The detailed analysis of this qualitative case study provides teacher educators with more resources and information regarding how teachers' mathematical knowledge affects their teaching practices and what knowledge early childhood teachers should develop to scaffold young children's talk and learning in mathematics.

For administrators: Freedom and support for improving teachers' practices

This study also offers implications for administrators about how to support early childhood teachers' growth, learning, and their practices. A school's administrator needs to provide early childhood teachers with instructional freedom. According to Goldstein (2007), this freedom allowed by the principals could "give their teachers some degree of instructional autonomy; and this autonomy contributes to [teachers'] ability to use developmentally appropriate practices to teach the standards" (p. 47). This study showed that the participant teacher was granted certain freedom, by her principal, to choose what to teach and how to teach in her own ways. This freedom enabled her to plan, organize, and teach mathematics content, based on her professional beliefs of how young children develop and learn, in that she thought that the educational goals in kindergarten should be "fundamentally different from upper elementary and secondary school classrooms" (Cuban, 1992, p. 173). Such freedom also allowed her to be free from the pressure from the first- and second-grade teachers who usually wanted kindergarten teachers to focus more directly on instructing academics skills. This study indicates that instructional freedom by administrators is one of the factors attributed to deciding how early childhood teachers skillfully plan and facilitate mathematical discussion within mathematics curriculum and timeline, along with its scope and sequence.

The school district's administrators also should provide early childhood teachers with professional development programs. This study reveals the participant teacher to be skillful and excellent at creating a respectful learning environment and at actively utilizing various instructional strategies to scaffold young children's talk and learning. She faced a challenge, however, in improving her own teaching strategies at helping children's engagement in mathematical discussion:

> We try to go to staff developments, but many times the district will give us staff development, so that, are for elementary school, but we are at the bottom end, you know, at the very beginning and many times they don't have math staff development geared for, like, kindergarten students or early childhood students, so that's one of the big problems (Interview transcript, 05/19/11).

This shows that her school district did not provide mathematics staff development programs for kindergarten teachers, and thus she did not have opportunities to examine, rebuild, and improve her own teaching practices of mathematics. This suggests that administrators need to offer professional development programs, seminars, or workshops that should target early childhood mathematics education and be more focused on changing and improving early childhood teachers' knowledge and practices in mathematics (Richardson & Placier, 2001). Moreover, the participant teacher, as an experienced teacher, had fewer opportunities to reflect on her own teaching practices. Indeed, she had a fixed daily schedule to follow, as well as various requirements such as meeting the curriculum of school district and state standards. For this, according to a report by Bloom (2008), administrators should consider how to help create professional learning communities of teacher discussion and planning groups consisting of colleagues. Supporting systems in a school such as inquiry groups and collaborative groups must be needed in changing mathematics classroom practices aimed at teaching for understanding (Tittle, 2006). In doing so, administrators help early childhood teachers mature into expert teachers able to scaffold young children's attempts at mathematical ways of speaking and thinking (Anthony & Walshaw, 2009).

LIMITATIONS AND FURTHER STUDY

Using observation, visual analysis, semi-structured interviewing, and various forms of documentation, this four-month qualitative case study detailed how one particular kindergarten teacher tried to help children enhance their mathematical discussions. It also explored certain instructional strategies that helped the teacher scaffold and expand children's capacity for mathematical conversation, discussion and curiosity. However, the investigation has several limitations, and these imply many possible future research directions. These would be in the area of what it is that early childhood teachers actually do to deal with mathematical discussion and that they should do for young children's mathematical learning and development.

First, this study employed a single-case design. My research aimed to gain an in-depth understanding of one particular case within the specific situation of this case involved. The single-case design is well suited to investigating an individual teacher's way of knowing and thinking about her role in mathematical discussion, as well as her professional strategies to orchestrate mathematical discussion for young children within today's public school systems. Nevertheless, the data presented in this study is not broad enough to provide a general understanding of the phenomenon (Yin, 2009). In this respect, future research should carry out a study using more than one case. This type of case study offers "a cross-case analysis suggesting generalizations" (Merriam, 1998, p. 40) about what constitutes a teacher's role in mathematical discussion. By looking at a range of similar and contrasting cases, the evidence from multiple cases is "often considered more compelling" (Yin, 2009, p. 53). Such data can enhance the external validity or generalizability of the findings of a case study to other situations (Merriam, 1998).

Second, this study relied on a limited period of data collection. In order to construct a rich and descriptive case study, I collected data over a period of 14 weeks, between February 2011 and May 2011. After I spent a total of close to 35 hours in the participant teacher's mathematics lessons, I felt I reached data "saturation" (Corbin & Strauss, 2008, p. 143). This overall time spent on this site and the number of visits was adequate to persistently observe and to understand the realities of a particular teacher's mathematics classroom as it exists (Merriam, 1998). Nonetheless, the value of data collected only from a spring semester is limited. What is still needed is gathering more data over a period of one academic year. Long-term observation at the site allows the researcher to obtain additional data on how, as the school year progressed, the teacher made different efforts at mathematical discussion (e.g., Sherin, 2002).

Third, the analysis of the data in this qualitative case study followed a process grounded in the constant-comparative method (Merriam, 1998). Such a strategy provided me with a step-by-step way directing how to construct meaning from qualitative data. This sequential approach to data collection and analysis also enabled me “to identify relevant concepts, follow through on subsequent questions, and listen and observe in more sensitive ways” (Corbin & Strauss, 2008, p. 57). In doing so, I could gain in-depth understanding of how a participant teacher played a role in creating a learning environment for respectful talk, in planning a discussion-intensive mathematics lessons, in scaffolding young children’s talk and learning, as well as what challenges she faced in enacting mathematical discussion in her everyday mathematics lessons. However, the finding of this study did not include what kinds of a teacher’s actions and words in mathematical discussion actually affect young children’s understanding and achievements in mathematics. In this respect, more research is needed for the analysis of audio-recorded conversations between the teacher and students by employing the method of discourse analysis (Cazden, 2001; Erickson, 2004; Mercer, 2004).

Last, my primary research focus was on understanding how an experienced teacher represents her conceptions, experiences, and decisions for discussion-intensive mathematics lessons through her own language and actions. In the current study, I directly looked at a teacher's instructional strategies to initiate and scaffold student discussions in the path of their mathematics learning. However, as I illustrated in Chapter Two, I, as a researcher, acknowledge that various factors of both students and teachers' cultural, ethnical, and linguistic backgrounds can affect the classroom discussion as a social context of learning and teaching mathematics. Future inquiry is needed to identify what specific aspects of classroom discussions can be highly related to the processes of school success and failure of ethnic and linguistic minority students (Edelsky, 2006), as well as to explore how the teacher's own cultural and linguistic backgrounds can influence their teaching practices of interacting and communicating with students during mathematics lessons.

Appendix A. Schedule of the data collection process

Phase I: Gaining entry into the field				
Week 1	Pilot classroom observation Initial interview	• Checking the appropriate place to set up the microphones in the classroom • Gathering the demographic information of the participant		
Phase II: Data collection and member checking				
Data collection	Observing math lessons Taking Fieldnotes Reflexive journals Audio-recording specific times Informal conversations Documenting	Interviewing	Member checking	Preliminary Analysis
Week 2				
Week 3		1st interview		
Week 4				
Week 5		2nd interview	Share notes	
Week 6				
Week 7		3rd interview		
Week 8				
Week 9		4th interview	Share notes	
Week 10				
Week 11		5th interview		
Week 12				
Week 13		6th interview	Share notes	
Phase III: Follow-up interview and exiting from the field				
Week 14	Until to reach data "saturation" (Strauss & Corbin, 1998)	Follow-up interview	Share notes Peer examination	

Appendix B. Standard Interview Protocol[12]

Gather demographic information from the teacher:

- Could you tell me about your educational experiences?
- How long have you been teaching?
- How long have you been teaching in this school?
- Which grade have you been teaching?
- How long have you been teaching kindergarten?

Ask about the teacher's past experience with math:

- Would you talk about your experience learning math?
- What do you remember about learning math in elementary school?
- When you were young, did you like math? Why did you like or not like math?
- How do you think your past experiences with math influence your math teaching?

Discuss the teacher's math lessons:

- Please tell me about your math lessons up to this point.
- How do you plan math lessons for your students?
- What are the main steps you take while designing a math lesson?
- How do you incorporate state standards or your school district curriculum into your math instruction?
- What do you focus on in teaching math for your kindergarten students?
- In your perspective, what is effective math instruction for your students?

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[12] A list of possible interview questions was developed based on the studies of O'Connell and O'Connor (2007) and Truxaw, Gorgievski, and DeFranco (2008).

Ask about the teacher's understanding of mathematical discussion:

- In your understanding, how would you describe the role of classroom discussion in teaching and learning math, in general and/or in your particular classroom?
- Do you think mathematical meanings can be negotiated in different ways? Why or why not?
- What role does classroom discussion play, if any, in children's math learning and development? If any, what are some benefits of asking students to discuss their math ideas with each other?
- Do you think classroom discussion affects young children's math achievement? Why or why not? Can you give me specific examples?
- Do you think classroom discussion affects your math lesson plans or your teaching practices? Assuming it does, how does mathematical discussion benefit you?
- How does listening to students' math thinking, either oral or written, help you refine your teaching methods?

Discuss the teacher's conception of his/her role in mathematical discussion:

- In your perspective, how would you describe your role in orchestrating children's engagement in mathematical discussion?
- Do you initiate children's rights and responsibilities as participants in discussion? How and why do you accomplish this?
- What is your role in conveying exact math concepts to children?
- What role should you play when asking thoughtful questions and responding to students' math ideas?
- How do you explain your role in helping children listen better and become better presenters during mathematical discussion?

- Could you describe your role in providing children with opportunities that enable children to debate alternative approaches to math problems and to share their thoughts and methods with each other?
- How do you explain your role in helping children argue and defend their own positions in discussion?
- Do you think your role in classroom discussion is important to promote children's mathematical thinking and development? Why or why not?
- What strengths and/or weaknesses do you have in facilitating math discussion?

Ask about the teacher's instructional strategies:

- In your mathematics lesson, how do you encourage classroom discussion?
- How do you give children a chance to engage in mathematical discussion?
- How do you define and clarify participants' rights and responsibilities in mathematical discussion?
- How do you create a respectful, trusting, comfortable, and nonthreatening climate for classroom discussion?
- What strategies do you use to enable children to carefully listen to others' explanations about their reasoning?
- How do you help children explain or justify their mathematical ideas?
- How do you ask children to verbally repeat the steps of mathematical procedures?
- In what ways do you ask children for alternative strategies in your math class?
- How do you ask children to verbally elaborate the results of their thinking to others?
- How might you encourage children to share their thoughts and methods with each other?
- What strategies do you use to help individual or group presentations benefit children, both the speakers and listeners?
- What types of questions do you use to develop and refine students' thinking?

- What strategies do you have to respond to students' wrong answers?
- What strategies do you use to accept all students' ideas?
- In what ways do you plan math activities for children who struggle with talking about math?
- What types of problem-based tasks do you use to stimulate students' engagement in mathematical discussion?
- How do you promote mathematical discussion through the physical arrangement of the classroom?
- What strategies do you use for facilitating small group discussions during math lessons?
- How do you ensure that all children participate in discussion in your math class?

Discuss the challenges facing the teacher in orchestrating mathematical discussion:

- What challenges do you experience when integrating math mandatory standards in your teaching practices? How do you handle these challenges?
- What are the challenges you face when you consider mathematical discussion in your math lesson plans?
- What are the challenges you face in creating mathematical discussion in your particular classroom or/and in your particular school context? If any, can you give specific examples?
- What challenges do you face in improving your own teaching strategies at helping children's engagement in mathematical discussion?
- Do you have the opportunities to collaborate with colleagues to discuss and expand your understanding of mathematical discussion? When working with your colleagues, what challenges do you face? How do you manage these challenges?

Prompts for semi-structured interviews:

- You said that ______. Am I getting that right?
- Would you clarify ______ you talked about in the interview?
- Is there anything else you would like to say, or do you any questions you wish I had asked?

Prompts for follow-up questions after observing a specific math lesson:

- At that time, you showed the ______ activity. What intention did you have for preparing this activity?
- During interacting with your students, what did you focus on?
- In ______ case, what do you think is your role for your students?
- When you ______, what special purpose did you have for it?
- In ______ activity, I observed ______. Why do you think it is important?
- In ______ situation, how/why did you decide to do that?

References

Adler, P., & Adler, P. (1994). Observational techniques. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 377-392). Thousand Oaks, CA: Sage.

Ageyev, V. S. (2003). Vygotsky in the mirror of cultural interpretations. In Kozulin, A., Gindis, B., Ageyev, V. S., & Miller, A. M. (Eds.), *Vygotsky's educational theory in cultural context* (pp. 432-449). Cambridge, UK: Cambridge University Press.

Amos, S. F. (2007). Talking mathematics. *Teaching Children Mathematics, 14*(2), 68-73.

Anthony, G., & Walshaw, M. (2009). *Effective pedagogy in mathematics*. Belley, France: International Academy of Education.

Ball, D. L., & Friel, S. N. (1991). Implementing the "professional standards for teaching mathematics": What's All This Talk about "Discourse"? *The Arithmetic Teacher, 39*(3), 44-47.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Banks, J. A. (1998). The lives and voices of researchers: Implications for education citizens in a multicultural society. *Educational Researcher, 27*, 4-17.

Baxter, J., Woodward, J., & Olson, D. (2001). Effects of reform-based mathematics instruction on low achievers in five third-grade classrooms. *Elementary School Journal, 101,* 529–549.

Bloom, M. (2008). Happy teachers tied to good students, Austin American-Statesman, Wednesday, November 12, 2008.

Bodrova, E., & Leong, D. (2007). *Tools of the mind: The Vygotskian approach to early childhood education*. Upper Saddle River, NJ: Prentice Hall.

Bonk, C. J., & Cunningham, D. J. (1998). Searching for learner-centered, constructivist, and socio-cultural components of collaborative educational learning tools. In C. J. Bonk & K. S. King (Eds.), *Electronic collaborators: Learner-centered technologies for literacy, apprenticeship, and discourse* (pp. 25-50). Mahwah, NJ: Erlbaum.

Borko, H., & Putnam, R. (1996). Learning to Teach. In D. Berliner & R. Calfee (Eds.), *Handbook of Educational Psychology* (pp. 673-708). New York: MacMillan.

Brandt, B. (1999). Recipients in elementary mathematics classroom interaction. In I. Schwank (ed.), *European Research in Mathematics Education* (Vol. 1: Internet-Version, pp. 308-319). Retrieved September 1, 2010, from http://www.fmd.uni-osnabrueck.de/ebooks/erme/cerme1-proceedings/cerme1-proceedings.html.

Bransford, J., Derry, S., Berliner, D., and Hammerness, K. (2005) Theories of learning and their roles in teaching. In L. Darling-Hammond and J. Bransford (eds.) *Preparing Teachers for a Changing World* (pg. 40-87). San Francisco: Jossey-Bass Publishers.

Brenner, M. (1998). Development of mathematical communication in problem solving groups by language minority students. *Bilingual Research Journal, 22*(2/4), 149-174.

Bruner, J. (1983). *Child's talk: Learning to use language*. Oxford: Oxford University Press.

Bussi, M. G. B., & Bartolini, G. (1998). Joint activity in mathematics classrooms: A Vygotskian analysis. In F. Seeger, J. Voigt and U. Waschescio (Eds.), *The culture of the mathematics classroom,* 13-49. Cambridge: Cambridge University Press.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L, Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Casa, T. M., & DeFranco, T. C. (2002). Examining the Nature and Role of Discourse in the Teaching of Mathematics: A Case Study of Two Preservice Teachers. Retrieved January, 14, 2010, from http://www.education.duq.edu/leaders/EQRE/papers2002/Casa&DeFranco.pdf

Cazden, C. (2001). *Classroom discourse: The language of teaching and learning* (2nd edition). Portsmouth, NH: Heinemann.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2003). *Classroom Discussions: Using Math Talk to Help Students Learn, Grades 1-6*. Sausalito, CA: Math solutions publications.

Chazen, D. & Ball, D. L. (2001). Beyond being told not to tell. *For the Learning of Mathematics, 19*(2), 2-10.

Cimbricz, S. (2002). State-mandated testing and teachers' beliefs and practice. *Education Policy Analysis Archives, 10*(2). Retrieved from http://epaa.asu.edu/epaa/v10n2.html.

Clark, C., & Peterson, P. (1986). Teachers' thought processes. In M. C.Wittrock (Ed.), *Handbook of research on teaching* (3rd ed., pp. 255–296). New York, NY: Macmillan.

Clements, D. H. (2004). Major themes and recommendations. In D. H. Clements, J. Sarama, & A-M. DiBiase. (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 7–72). Mahwah, NJ: Lawrence Erlbaum Associates.

Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking, and classroom practice. In E. Forman, N. Minick, & C. Stone (Eds.), *Contexts for* *learning: Sociocultural dynamics in children's development* (pp. 91–119). Oxford, UK: Oxford University Press.

Cobb, P., Yackel, E., Wood, T., Wheatley, G. & Merkely, G. (1998). Creating a problem solving atmosphere. *The Arithmetic Teacher, 36*(1), 46-47.

Cooke, B. D., & Buchholz, D. (2005). Mathematical communication in the classroom: A teacher makes a difference. *Early Childhood Education Journal, 32*(6), 365-369.

Copple, C. E. (2004). Mathematics curriculum in the early childhood context. In D. H. Clements & J. Sarama (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics instruction* (pp. 83-87). Mahwah, NJ: Lawrence Erlbaum Associates.

Copple, C., & Bredekamp, S. (2009). *Developmentally appropriate practice in early childhood programs serving children from birth through age 8* (3rd ed.). Washington, DC: National Association for the Education of Young Children.

Corbin, J., & Strauss, A. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (3rd ed.). Thousand Oaks, CA: Sage.

Crotty, M. (1998). *The foundations of social research: Meaning and perspective in the research process*. London: Sage.

Cuban, L. (1992). Why some reforms last: The case of kindergarten. *American Journal of Education, 100*(2), 166–194.

DiSessa, A. (2000). *Changing minds*. Cambridge, MA: MIT Press.

Do, S. L., & Schallert, D. L. (2004). Emotions and classroom talk: Toward a model of the role of affect in students' experiences of classroom discussion. *Journal of Educational Psychology, 96*, 619-634.

Edelsky, C. (2006). *With literacy and justice for all: Rethinking social in language and education* (2nd ed.). Mahwah, NJ: Lawrence Erlbaum Associates.

Elkind, D. (1987). *Miseducation: Preschoolers at Risk*. New York: Alfred A. Knopf.

Empson, S. (2003). Low performing students and teaching fractions for understanding: An interactions analysis. *Journal for Research in Mathematics Education, 34,* 305–343.

Engel, S. (2011). Children's Need to Know: Curiosity in Schools. *Harvard Educational Review, 81*(4). 625-645.

Erickson, F. (1996). Going for the zone: The social and cognitive ecology of teacher-student interaction in classroom conversations. In D. Hicks (Ed.), *Discourse, Learning, and schooling* (pp. 29-62). New York: Cambridge University Press.

Erickson, F. (2004). *Talk and Social Theory*. Malden, MA: Polity Press.

Erlandson, D. A., Harris, E., L., Skipper, B. L., & Allen, S. D. (1993). *Doing naturalistic inquiry: A guide to methods*. London: Sage.

File, N. (1995). Applications of Vygotskian theory to early childhood education: Moving toward a new teaching-learning paradigm. In S. Reifel (Ed.), *Advances in Early Education and Day Care* (Vol. 7, pp. 295-317). Greenwich, CT: JAI Press, Inc.

Floden, R. E. (2001). Research on effects of teaching: A continuing model for research on teaching. In V. Richardson (Ed.), *Handbook of research on teaching* (pp. 3–16). New York, NY: Macmillan.

Forman, E., & Ansell, E. (2001). The multiple voices of a mathematics classroom community. *Educational Studies in Mathematics, 46*, 114–142.

Forman, G. (1993). Constructivism. In J. Roopnarine & J. Johnson (Eds), *Approaches to early childhood education* (2nd ed., pp. 137-155). Columbus, OH: Merrill.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. Lester (ed.), *The second handbook of research on mathematics teaching and learning* (pp. 225-256). Reston, Va.: NCTM, 2007

Frankenstein, M. (1990). Incorporating race, gender, and class issues into a critical mathematical literacy curriculum. *Journal of Negro Education, 59*, 336-359.

Frost, J. L., Wortham, S., & Reifel, S. (2008). *Play and child development* (3rd ed.). Upper Saddle River, NJ: Prentice Hall/Merrill.

Ginsburg, H. P. (2006). Mathematical play and playful mathematics: A guide for early education. In D. Stinger, R. Golinkoff, & K. Hirsh-Patek (Eds.), *Play =learning: How play motivates and enhances children's cognitive and social-emotional growth* (pp.145-165). Now York: Oxford University Press.

Ginsburg, H. P. & Ertle, B. (2008). Knowing the mathematics in early childhood mathematics. In O. N. Saracho, & B. Spodek (Eds.) *Contemporary perspective on mathematics in early childhood education* (pp. 45-66). Charlotte, NC: IAP/Information Age Pub.

Glaser, B., & Strauss, A. (1967). *The discovery of grounded theory: Strategies for qualitative research*. Chicago: Aldine.

Goffman, E. (1981). *Forms of talk*. Philadelphia: University of Pennsylvania Press.

Goldstein, L. S. (1998). Caught in the middle: Tension and contradiction in enacting the *Primary grade curriculum*. *Curriculum Inquiry, 28*(3), 311–337.

Goldstein, L. S. (2007). Beyond the DAP versus standards dilemma: Examining the unforgiving complexity of kindergarten teaching in the United States. *Early Childhood Research Quarterly, 22,* 39-54.

Goodwin, C., & Goodwin, M. H. (2004). Participation. In A. Duranti (Ed.), *A companion to linguistic anthropology* (pp. 222-244). Oxford, UK: Blackwell.

Goodwin, M. H. (1990). *He-said-she-said: Talk as social organization among Black children*. Bloomington, IN: Indiana University Press.

Guba, E. G., & Lincoln, Y. S. (1998). Competing paradigms in qualitative research. In N. K. D. Y. S. Lincoln (Ed.), *The landscape of qualitative research* (Vol. 2, pp. 195-220). Thousand Oaks, CA: Sage.

Gutstein, E. (2003). Teaching and Learning Mathematics for Social Justice in an Urban, Latino School. *Journal for Research in Mathematics Education, 34*(1), 37-73.

Gutstein, E. (2006). *Reading and Writing the World With Mathematics: Toward a Pedagogy for Social Justice*. Routledge Farmer.

Hammerness, K., Darling-Hammond, L., & Bransford, J. (2005). How teachers learn and develop. In L. Darling-Hammond & J. Bransford (Eds.), *Preparing teachers for a changing world* (pp. 358–389). San Francisco, CA: Jossey-Bass.

Hanline, M. F., Milton, S., & Phelps, P. C. (2008). A longitudinal study exploring the relationship of representational levels of three aspects of preschool sociodramatic play and early academic skills. *Journal of Research in Childhood Education, 23*(1), 19-28.

Henningsen, M. & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*(5), 524-549.

Hiebert, J., & Wearne, D., (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal, 30,* 393–425.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., Oliver, A., and Human, P. (1997) The nature of classroom tasks. In Hiebert, et. al., *Making Sense: Teaching and learning mathematics with understanding* (pp. 17-28*)*. Portsmouth, NH: Heinemann.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008a). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372-400.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008b). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26*(4), 430-511.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

John-Steiner, V. & Mahn, H. (1996). Sociocultural approaches to learning and development: A Vygotskian Framework. *Educational Psychologist, 31*(3/4), 191-206.

Johnson, B., & Christensen, L. (2007). *Educational research: Quantitative, qualitative, and mixed approaches*. Thousand Oaks, CA: Sage.

Jung, H. Y., & Reifel. S. (2011). Promoting children's communication: A kindergarten teacher's conception and practice of effective mathematics instruction. *Journal of Research in Childhood Education, 25*, 194-210.

Kahle, J.B. (1996). *Thinking about equity in a different way*. Washington, DC: The American Association for the Advancement of Science.

Kamii, C. & Ewing, J. K. (1996). *Basing teaching on Piaget's constructivism*. Childhood Education.

Kamii, C. & Kato, Y. (2006). Play and mathematics at age one to ten. In D. P. Fromberg and D. Bergen (Eds.), *Play from birth to twelve: contexts, perspectives, and meanings* (2nd Ed.), 187-198. New York: Routledge.

Kamii, C., & Anderson, C., (2003). Multiplication Games: How we made and used them. *Teaching Children Mathematics, 10* (3), 135-141.

Kamii, C., Miyakawa, Y., & Kato, Y. (2004). The development of logico-mathematical knowledge in a block-building activity at ages 1-4. *Journal of Research in Childhood Education, 19*(1), 14.

Khisty, L. L. (1995). Making inequality: Issues of language and meanings in mathematics teaching with Hispanic students. In W. Secada, E. Fennema, & L. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 279–297). Cambridge, UK, New York, NY: Cambridge University Press.

Labinowicz, E. (1985). *Learning from children: new beginnings for teaching numerical thinking: a Piagetian approach*. Menlo Park, CA: Addison-Wesley.

Ladson-Billings, G. (2009). *The Dreamkeepers: Successful Teachers of African American Children* (2nd ed.). New York: Wiley & Sons.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*, 29–63.

Lampert, M., & Blunk, M. (Eds.). (1998). *Talking mathematics in school: Studies of teaching and learning*. Cambridge, UK: Cambridge University Press.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. New York: Cambridge University Press.

Lee, J. S., & Ginsburg, H. P. (2009). Early childhood teachers' misconceptions about mathematics education for young children in the United States. *Australasian Journal of Early Childhood, 34*(4), 37-45.

Lee. J. S., & Ginsburg, H. P. (2009). Early childhood teachers' misconceptions about mathematics education for young children in the United States. *Australasian Journal of Early Childhood. 34*(4). 37-45.

Leikin, R., & Dinur, S. (2007). Teacher flexibility in mathematical discussion. *Mathematical behavior, 26,* 328-347.

Lubienski, S. T. (2002). Research, reform, and equity in U.S. mathematics education. *Mathematical Thinking and Learning, 4,* 103–125.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum.

Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States* (2nd ed.). Mahwah, NJ: Erlbaum.

McClain, K., & Cobb, P. (2001). An analysis of development of sociomathematical norms in one first-grade classroom. *Journal for Research in Mathematics Education, 32,* 236–266.

McIntyre, E., Kyle, D., & Moore, G. (2006). A Primary-Grade Teacher's Guidance Toward Small-Group Dialogue. *Reading Research Quarterly, 41*(1), 36-66.

McIntyre, E., Kyle, D., Moore, G., Sweazy, R. A., & Greer, S. (2001). Linking home and school through family visits. *Language Arts, 78*(3), 264-272.

Mercer, N. (1995). *Guiding the construction of knowledge: Talk amongst teachers and learners*. Clevedon, UK: Multilingual Matters, Ltd.

Merriam, S. (1998). *Qualitative research and case study applications in education*. San Francisco, CA: Jossey-Bass.

Merriam, S., Johnson-Bailey, J., Lee, M., Kee, Y., Ntseane, G., & Muhamad, M. (2001). Power and positionality: Negotiating insider/outsider status within and across cultures. *International Journal of Lifelong Education, 20*(5), 405-416.

Mertens, D. (2005). *Research and evaluation in education and psychology: Integrating diversity with quantitative, qualitative, and mixed methods*. Thousand Oaks, CA: Sage.

Meyer, D. K., & Turner, J. C. (2007). Scaffolding emotions in classrooms. In P.A. Schutz & R. Pekrun (Eds.), *Emotion in education* (pp. 243-258). San Diego: Elsevier Inc.

Mistertta, R. M. (2008). *Teachers engaging parents and children in mathematical learning*. Maryland: Rowman & Littlefield Education.

Moje, E. B., & Lewis, C. (2007). Examining opportunities to learn literacy: The role of critical sociocultural literacy research. In C. Lewis, P. Enciso, & E. B. Moje (Eds.), *Reframing sociocultural research on literacy: Identity, agency, and power* (pp. 15-48). Mahwah, NJ: Erlbaum.

Moll, L. C. (1990). *Vygotsky and education: Instructional implications of sociohistorical psychology*. New York: Cambridge University Press.

Moll, L. C. (2001). Through the Mediation of Others: Vygotskian Research on Teaching. In V. Richardson (Ed.), *Handbook of Research on Teaching* (pp. 111-129). New York: MacMillan.

Mukhopadhyay, Powell, & Frankenstein, (2009). An ethnomathematics perspective on culturally responsive mathematics education. In G. Brian (ed.), *Culturally responsive mathematics education* (pp. 65-84). New York: Routledge.

Nathan, M. J., Elliott, R., Knuth, E., & French, A. (March 1997). *Self-reflection on teacher goals and actions in the mathematics classroom*. Paper presented at the annual meeting of the American Educational Research Association annual meeting, Chicago.

Nathan, M., & Knuth, E. (2003). A study of whole classroom mathematical discourse and teacher change. *Cognition and Instruction, 21,* 175–207.

National Association for the Education of Young Children and National Council of Teachers of Mathematics (2002). *Early childhood mathematics: Promoting good beginnings*. Joint position statement. Washington, DC: NAEYC.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics*. Reston, VA: Author.

National Research Council. (2000). *How people learn: Brain, mind, experience, and school*. Washington, DC: National Academy Press.

Nicol, C., & Crespo, S. (2005). Exploring mathematics in imaginary places: Rethinking what counts as meaningful contexts for learning mathematics. *School Science and Mathematics, 105*(5), 240-251.

O'Connell, S., & O'Connor, K. (2007). *Introduction to communication: Grades preK-2. The Math Process Standards Series*. Portsmouth, NH: Heinemann.

O'Connor, E., Fish, M., & Yasik, A. (2004). The influence of teacher experience on the elementary classroom system: An observational study. *Journal of Classroom Interaction, 39*(1), 11-18.

O'Connor, M. C. (1998). Language socialization in the mathematics classroom: Discourse practices and mathematical thinking. In M. Lampert & M. L. Blunk (Eds.), *Talking mathematics in school: Studies of teaching and learning* (pp. 17-55). New York: Cambridge University Press.

O'Connor, M. C. (2001). "Can any fraction be turned into a decimal?" A case study of the mathematical group discussion. *Educational Studies in Mathematics, 46,* 143–185.

O'Connor, M. C., & Michaels, S. (1996). Shifting participant frameworks: Orchestrating thinking practices in group discussion. In D. Hicks (Ed.), *Discourse, learning and schooling* (pp. 63–103). New York: Cambridge University Press.

Patton, M. Q. (1990). *Qualitative evaluation and research methods*. Newbury Park, CA: Sage.

Patton, M. Q. (2002). *Qualitative evaluation and research methods: A systematic approach* (3rd ed.). CA: Sage.

Piaget, J. (1953). The origin of intelligence in the child. London: Routledge.

Pierson, J., Maldonado, L. A., and Pierson, E. (2007). Talking Mathematics: A case study of one kindergarten teacher's practices to scaffold mathematical discourse. Paper presented at the annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, University of Nevada, Reno, Reno, Nevada.

Pintrich, P. R. (2003). A motivational science perspective on the role of student motivation in learning and teaching contexts. *Journal of Educational Psychology, 95*, 667-686.

Pirie, S., & Schwarzenberger, R. (1988). Mathematical discussion and mathematical understanding. *Educational studies in mathematics, 19*(4), 459-470.

Powell, D. R. (1988). *Parent education as early childhood intervention, emerging directions in theory, research and practice*. Albex Publishing Co.: New Jersey.

Richardson, V & Placier, P. (2001) Teacher Change. In V. Richardson (Ed.), *Handbook of Research on Teaching* (pp. 905-950). New York: MacMillan.

Rittenhouse, P. S. (1998). The teacher's role in mathematical conversation: stepping in and stepping out. In M. L. Blunk (Ed.), *Talking mathematics in school: Studies of teaching and learning* (pp. 163-189). New York: Cambridge University Press.

Rosiek, J. (2003). Emotional scaffolding: An exploration of the teacher knowledge at the intersection of student emotion and the subject matter. *Journal of Teacher Education, 54*(5), 399-412.

Rosiek, J., & Beghetto, R. (2009). Emotional Scaffolding: The Emotional and Imaginative Dimensions of Teaching and Learning. In P. A. Schutz & M. Zembylas (Eds.), *Advances in Teacher Emotion Research* (pp. 175-194). New York: Springer.

Rudd, L. C., Lambert, M. C., Satterwhite, M., & Zaier, A. (2008). Mathematical language in early childhood setting: What really counts? *Early Childhood Education Journal, 36*, 75-80.

Schallert, D. L., & Martin, D. B. (2003). A psychological analysis of what teachers and students do in the language arts classroom. In J. Flood, D. Lapp, J. R. Squire, & J.

M. Jensen (Eds.), *Handbook of research on teaching the English language arts* (2nd ed., pp. 31-45). New York: Macmillan.

Schwartz, S. L. (2005). *Teaching young children mathematics*. Westport, CT: Praeger.

Schwartz, S. L., & Brown, A., E. (1995). Communicating with young children in mathematics: A unique challenge. *Teaching Children Mathematics, 1*(6), 250-254.

Sherin, M. G. (2002). A balancing act: Developing a discourse community in a mathematics classroom. *Journal of Mathematics Teacher Education, 5*, 205-233.

Sherin, M. G. (2008). Facilitating meaningful discussion of mathematics. *Mathematics teaching in the middle school*. *6*(2). 122-125.

Shulman, Lee. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-27.

Silver, E. A., & Smith, M. S. (1996). Building discourse communities in mathematics classrooms: A worthwhile but challenging journey. In P. C. Elliott (Ed.), *Communication in mathematics, K-12 and beyond: 1996 yearbook* (pp. 20-28). Reston, VA: National Council of Teachers of Mathematics.

Simon, A. M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26(*2), 114–145.

Simon, A. M. (1997). Developing new models of mathematics teaching: An imperative for research on mathematics teacher development. In E. Fennema & B. Scott-Nelson (Eds.), *Mathematics teachers in transition* (pp. 55-86). Mahwah, NJ: Erlbaum.

Sipe, L., & Constable, S. (1996). A chart of four contemporary research paradigms: Metaphors for the modes of inquiry. *Taboo: The Journal of Culture and Education, 1*, 153-163.

Skipper, E. L., & Collins, E. N. (2003). Making the NCTM standards user-friendly for child care teachers. *Teaching Children Mathematics, 9*(7), 421–427.

Somer, R., Michaels, S., O'Connor, M. C., Resnick, L. (2009). Guided construction of knowledge in the classroom. In B. Schwarz, T. Dreyfus, & R. Hershkowitz, (Eds.) *Transformation of knowledge through classroom* (pp. 105-129). New York, NY: Routledge.

Stahl, R. J. (1994, March). *The essential elements of cooperative learning in the classroom*. ERIC Digest. Bloomington, IN: ERIC Clearinghouse for Social Studies/Social Science Education. (ERIC Document Reproduction Service No. 370 881)

Stake, R. E. (1995). *The art of case study research*. Thousand Oaks, CA: Sage.

Stake, R. E. (2005). Qualitative case studies. In N. Denzin & Y. Lincoln (Eds.), *The handbook of qualitative research* (3rd ed., Vol. 3, pp. 443-466). Thousand Oaks, CA: Sage.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning 10*, 313-340.

Tate, W. F. (1997). Race-ethnicity, SES, gender, and language proficiency trends in mathematics achievement: An update. *Journal for Research in Mathematics Education*, 28(6), 652–679.

Texas Education Agency (2011). Texas Essential Knowledge and Skills for Mathematics: Subchapter A. Elementary. Retrieved August 10, 2012, from http://www.tea.state.tx.us/index2.aspx?id=2147499971.

Texas Education Agency (n.d.a). 2010-2011 Academic Excellence Indicator System report. Retrieved August 10, 2012, from http://ritter.tea.state.tx.us/perfreport/aeis/2011/index.html

Texas Education Agency (n.d.b). Testing & Accountability. Retrieved August 10, 2012, from http://www.tea.state.tx.us/index.aspx?id=2147495410&menu_id=660&menu_id2=795&cid=2147483660

Tittle, C. (2006). Assessment of teacher learning and development. In P. Alexander and P. Winne (Eds.), *Handbook of Educational Psychology* (pp. 953-980). Mahwah, NJ: Lawrence Erlbaum Associates.

Tobin, K. G. (1986). Effects of teacher wait time on discourse characteristics in mathematics and language arts classes. *American Educational Research Journal, 23*(2), 191-200.

Tobin, K. G., & Capie, W. (1983). The influence of wait time on classroom learning. *European Journal of Science Education, 5(*1), 35-48.

Truxaw, M. P., Gorgievski, N., & DeFranco, T. C. (2008). Measuring K-8 teachers' perceptions of discourse use in their mathematics classes. *School Science and Mathematics, 108*(2), 58-70.

Vacc, N. N. (1993). Implementing the "professional standards for teaching mathematics": Teaching and learning mathematics through classroom discussion. *The Arithmetic Teacher, 41*(4), 225-227.

Valsiner, J. (1987). *Culture and the development of children's action: A cultural-historical theory of development*. Chichester, UK: Wiley.

Varol, F., & Farran. C. (2006). Early mathematical growth: how to support young children's mathematical development. *Early Childhood Education Journal*. *33*(6). 381-387.

Vygotsky, L. S. (1978). *Mind in society: The development of higher mental processes*. Cambridge, MA: Harvard University Press.

Vygotsky, L. S. (1981). The instrumental method in psychology. In J. V. Wertsch (Ed.), *The concept of activity in Soviet psychology* (pp. 134-144). Armonk, NY: Sharpe.

Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research, 78*(3), 516-552.

Wertsch, J. V. (1991). A sociocultural approach to socially shared cognition. In L. B. Resnick, J. M. Levine, & S. D. Teasley (Eds.), *Perspectives on socially shared cognition* (pp. 885-100). Washington, DC: American Psychological Association.

Wertsch, J. V. (1998). *Mind as action*. New York: Oxford University Press.

White, D. (2003). Promoting productive mathematical classroom discourse with diverse students. *Journal of Mathematical Behavior, 22,* 37–53.

Whitin, D. J., & Whitin, P. (2003). Talk counts: Discussing graphs with young children. *Teaching Children Mathematics, 10*, 142–149.

Wien, C. (2002). The press of standardized curriculum: Does a kindergarten teacher instruct with worksheets or let children play? *Canadian Children, 27*(1), 10–17.

Wien, C. (2004). *Negotiating standards in the primary classrooms: The teacher's dilemma*. New York, NY: Teachers College Press.

Wood, D. J., Bruner, J. S., and Ross, G. (1976). The role of tutoring in problem solving, *Journal of Child Psychology and Psychiatry, 17*(2), 89-100.

Woolfolk, A. (2004). *Educational psychology*. (9th ed.). Boston: Allyn and Bacon.

Worthington, M., & Carruthers, E. (2003). *Children's mathematics: Making marks, making meaning*. London, UK: Paul Chapman Publishing.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 2,* 458–477.

Yin, R. K. (2009). *Case study research: Design and methods* (4th ed.). Thousand Oaks, CA: Sage.

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